PS157 Home Experiment:
A Bouncing Ball

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1 Introduction:

When objects are dropped they don't all behave in the same way, we know that the properties of an object have impact how they will react to the collision they have with the ground, for example when we drop a book and a wine glass, regardless of the height which the objects are dropped at, we know they will behave in completely different ways. One of the ways in which objects that are dropped differ lies with whether they behave elastic or Inelasticly to the collision. This is the purpose of the coefficient of restitution, it is a numerical value that refers to how elastic or inelastic an object reacts to a collision, an object with an extremely low almost 0 coefficient of restitution will immediately collide with the ground and have almost no other motion, whereas an object with an extremely high coefficient of almost 1 will instead carry out another motion after the collision. A good example of an object with an extremely high coefficient of restitution is an ordinary bouncy ball, the defining characteristic of a bouncy ball is its ability to collide with a surface and immediately result in movement away from the surface.

The purpose of this experiment is to use a bouncy ball and discover it's coefficient of restitution, before making definitive claims about what the coefficient of restitution in an object is dependent upon. The first part of this experiment we will do by dropping the bouncy ball from predetermined heights and recording the height at which the ball reaches after its first bounce, with this data we can determine the coefficient of restitution using the equation:

$$r = \frac{v_1}{v_0} \tag{1}$$

Where r = coefficient of restitution, v_1 is the speed after the first bounce and v_0 is the speed at which the ball was dropped at.

$$r = \left(\frac{h_1}{h_0}\right)^{\frac{1}{2}} \tag{2}$$

For the second method we want to determine whether or not the coefficient of restitution is dependent on the number of times we bounce the bouncy ball. To do this we are using the equation:

$$ln(h_n) = 2n\ ln\ r + ln(h_0) \tag{3}$$

Where n = number of bounces, r = coefficient of restitution and h_0 is the original height the ball was dropped from.

In this experiment we will be graphing $ln(h_n)$ vs $2n$, which will give us a slope of lnr, in order to find the coefficient of restitution we will then use:

$$r = e^m \tag{4}$$

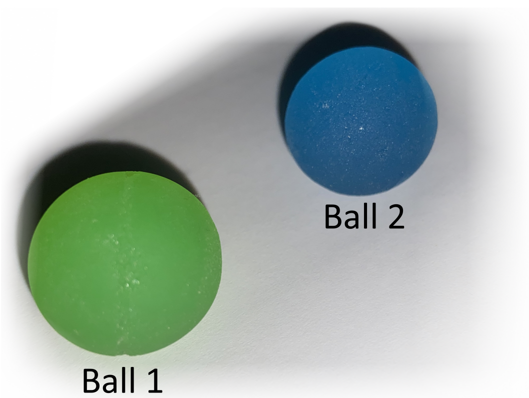
where m will be the slope of the graph.

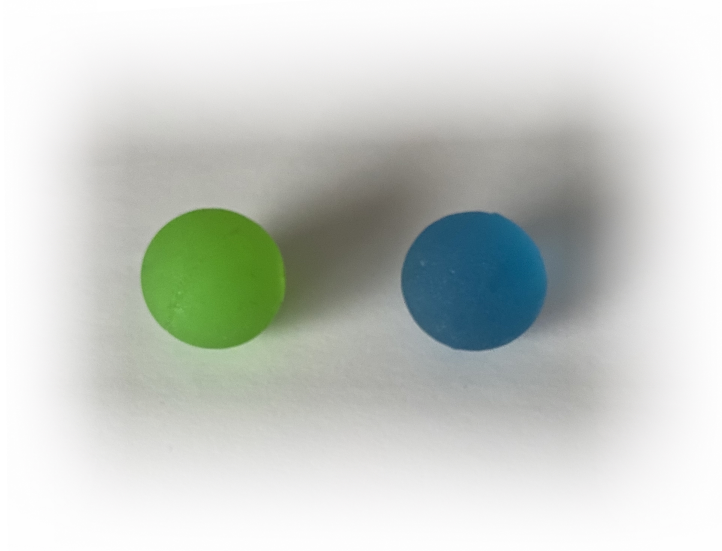
2 Method Experimental Set-up:

This experiment was carried out using a meter stick and two similar bouncy balls [Fig. 2.1] that will be labelled Ball 1 and Ball 2 for the purpose of this lab report. The initial set up of this experiment was simply placing the meter stick down on a hard wood surface and laying it flat against a wall, this was done so it would be easy to observe the heights reached by the bouncy balls over the course of the experiment, as we would need to record not only the height the bouncy balls were being dropped from, but also the subsequent heights reached by the bouncy ball after they were dropped.

The first part of the experiment (labelled method 1), involved collecting data points for initial drop height h_0 as well as the height achieved by each bouncy ball after one bounce h_1. It was decided that the drop heights would be carried out over the range of 1 meter and in intervals of 10cm, this would give us 10 data points to work with, which was deemed to be acceptable to achieve an accurate graph and coefficient value. Although each interval only called for two data points to be recorded- a number of test drops were carried out before getting the actual value, this was due to the maximum bounce height (h_1) being difficult to observe. Test drops would increase accuracy as it was beneficial to know where on the meter stick the bouncy ball was likely to reach and we could avoid focusing on the wrong space. (These test drops were omitted when tabulating our values.)

For the second part of the experiment (labelled method 2), a similar process to method 1 was carried out although this time the bouncy ball was allowed to bounce a number of times n which would be recorded, along with the bounce height achieved with this number of bounces h_n. This method shows that the coefficient of restitution isn't actually dependant on the drop height, as we can still accurately determine the coefficient of restitution for the bouncy balls without using the initial drop height.





. Figure 2.1: Bouncy Balls

3 Results:

Method 1:

Bouncy Ball 1

Original height (h_0) (m)	Bounce height (h_1) (m)	r $(\frac{h_1}{h_0})^{\frac{1}{2}}$
1	0.58	0.76
0.9	0.53	0.77
0.8	0.48	0.77
0.7	0.43	0.78
0.6	0.37	0.79
0.5	0.32	0.8
0.4	0.26	0.81
0.3	0.20	0.82
0.2	0.14	0.84
0.1	0.08	0.89

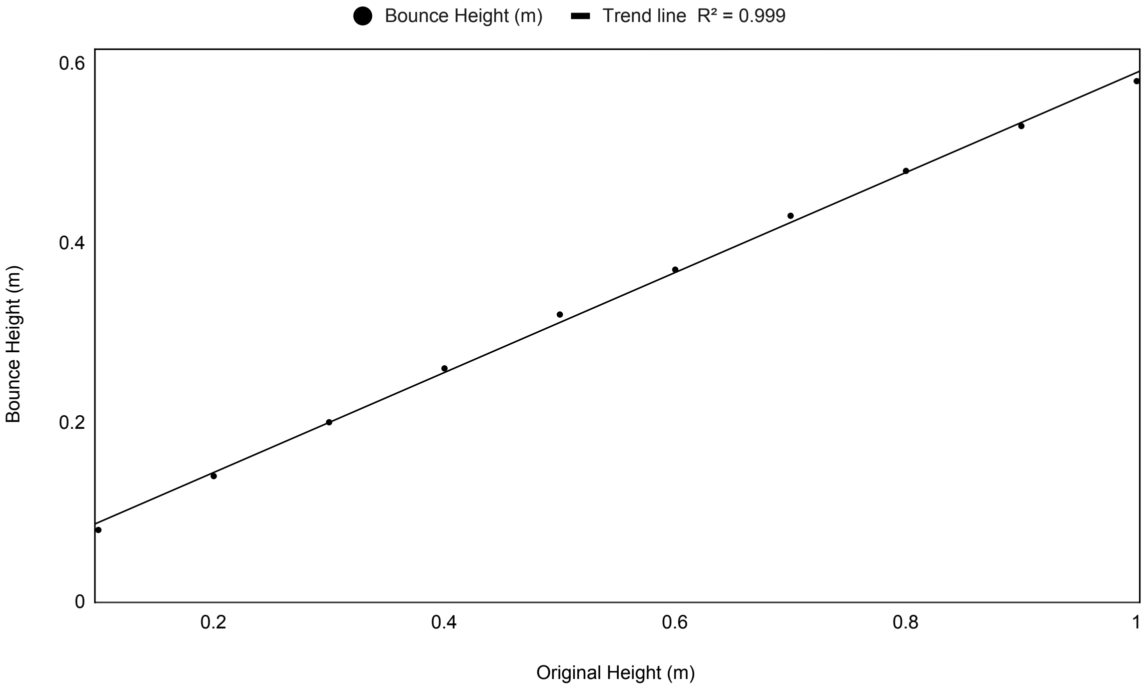


Figure 3.1: Bounce Height vs Original Height Graph (Bouncy Ball 1)

Using our equation of a slope:

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

We can determine the slope of our line

$$m = \frac{0.58 - 0.08}{1 - 0.1} = 0.\dot{5}$$

Finally, using Eq. 2 referenced in our introduction we can find the coefficient of restitution:

$$r = 0.\dot{5}^{(\frac{1}{2})} = \boxed{0.75 \pm 0.00741}$$

Bouncy Ball 2

Original height (h_0) (m)	Bounce height (h_1) (m)	r $(\frac{h_1}{h_0})^{\frac{1}{2}}$
1	0.69	0.83
0.9	0.64	0.84
0.8	0.57	0.84
0.7	0.49	0.84
0.6	0.42	0.84
0.5	0.36	0.85
0.4	0.28	0.84
0.3	0.22	0.86
0.2	0.15	0.87
0.1	0.09	0.95

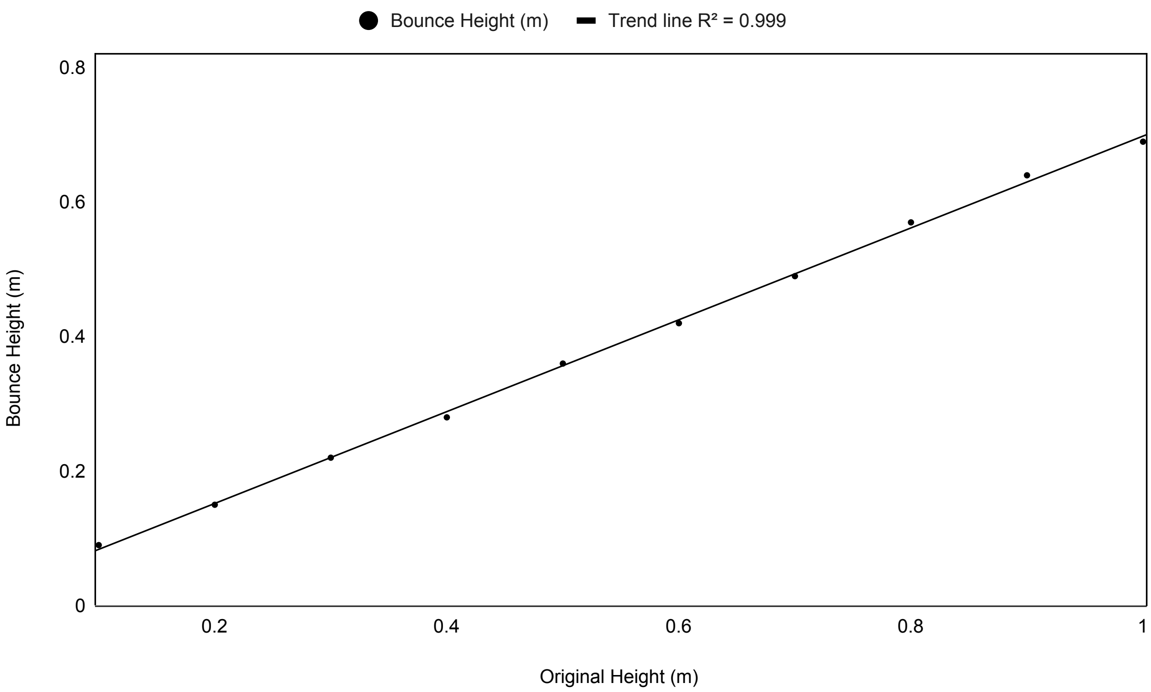


Figure 3.2: Bounce Height vs Original Height Graph (Bouncy Ball 2)

Using our equation of a slope:

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

We can determine the slope of our line

$$m = \frac{0.69 - 0.09}{1 - 0.1} = 0.\dot{6}$$

Finally, using Eq. 2 referenced in our introduction we can find the coefficient of restitution:

$$r = 0.\dot{6}^{(\frac{1}{2})} = \boxed{0.82 \pm 0.00781}$$

Method 2:

Bouncy Ball 1

Number of bounces (n)	2n	$\ln(h_n)$
1	2	-0.545
2	4	-0.844
3	6	-1.347

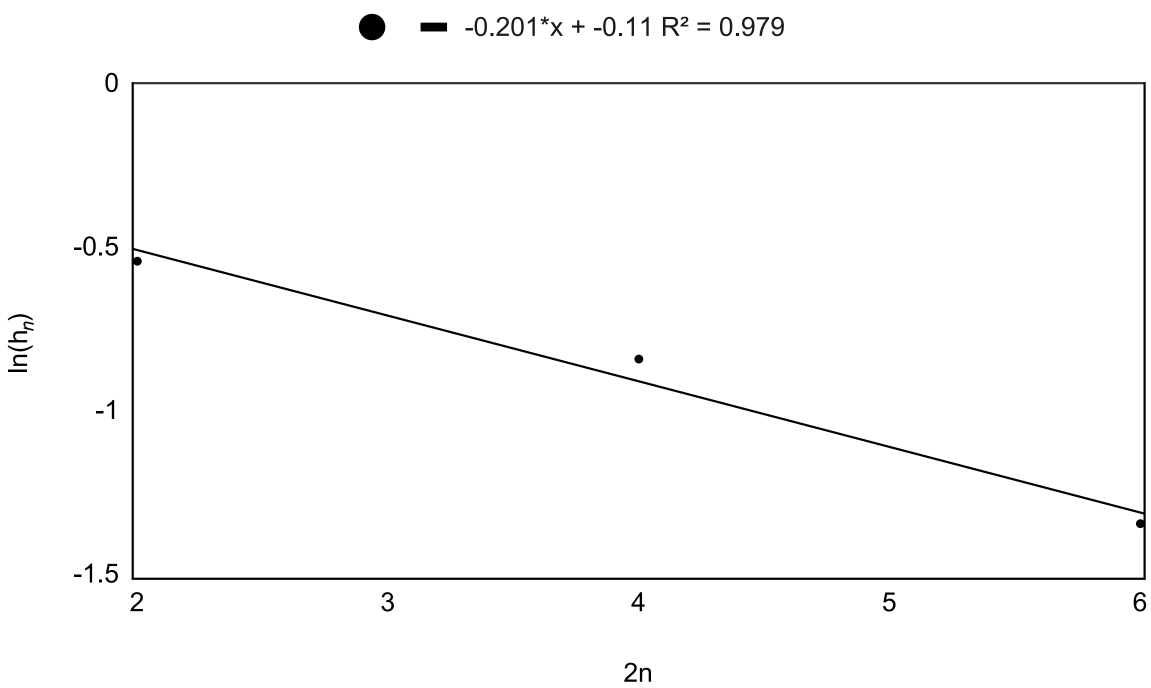


Figure 3.3: $\ln(h_n)$ vs 2n

Using our equation of a slope:

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

We can determine the slope of our line

$$m = \frac{-1.347 - (-0.545)}{6 - 2} = -0.2005$$

Finally, using Eq. 4 referenced in our introduction we can find the coefficient of restitution:

$$r = e^{-0.2005} = \boxed{0.82 \pm 0.02942}$$

Bouncy Ball 2

Number of bounces (n)	2n	ln(h_n)
1	2	-0.371
2	4	-0.868
3	6	-1.109

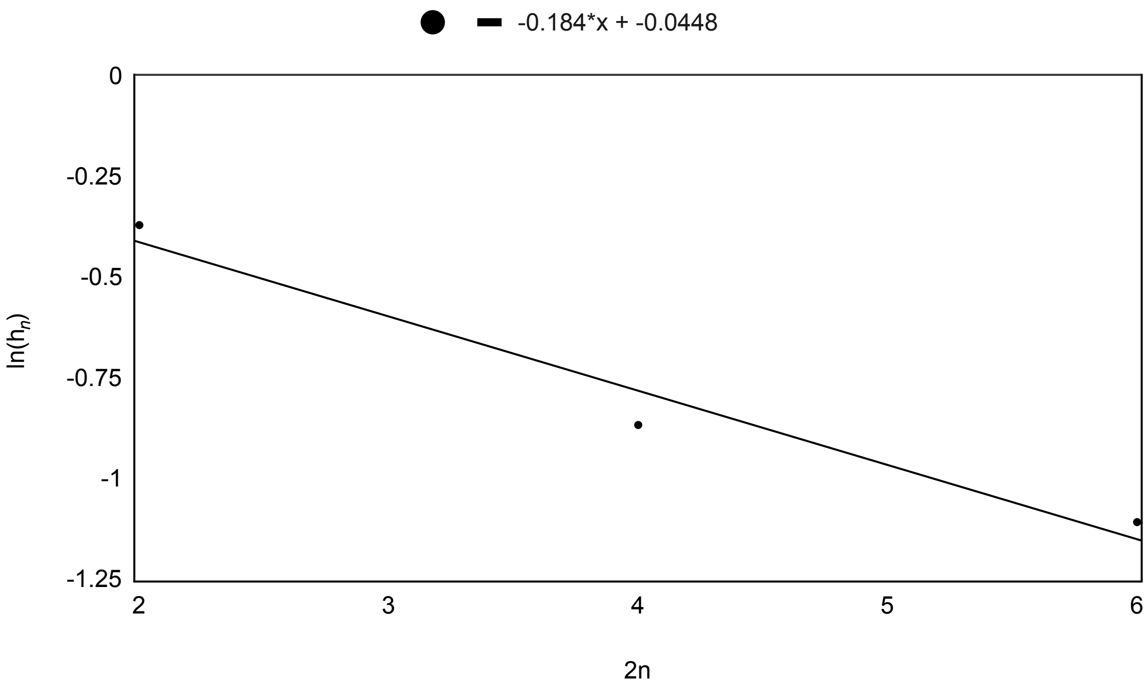


Figure 3.4: ln(h_n) vs 2n

Using our equation of a slope:

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

We can determine the slope of our line

$$m = \frac{-1.109 - (-0.371)}{6 - 2} = -0.185$$

Finally, using Eq. 4 referenced in our introduction we can find the coefficient of restitution:

$$r = e^{-0.185} = \boxed{0.83 \pm 0.03685}$$

4 Discussion:

While carrying out the experiment it became apparent that the method used to collect the results was by no means perfect and it was obvious that there was a lot of room for error. Although every effort was made to ensure that the ball's bounce height was being observed intently, due to the nature of objects in motion with a high velocity it is difficult for the human eye to be able to properly determine the maximum displacement of the ball from the ground when v = 0 in a fraction of a second. It is also unreasonable to expect that a human can correctly drop the ball from the exact same point in order to ensure maximum accuracy across the results. Although our line of best fit had R^2 values of 0.999 and 0.979, the difficulty felt when attempting to collect the values made it apparent how useful more scientific equipment would be in this type of scenario.

If this experiment was to be repeated with the goal of increasing the accuracy of the findings, it would be recommended that the help of light gates be used in order to accurately determine if the bouncy ball actually passes a certain height after one or multiple bounces. A platform which could drop the bouncy ball would be helpful in ensuring that the drop height remains consistent during the course of the experiment as well as preventing any extra energy being given to the ball as it is dropped towards the ground.

Another effective method of reducing inaccuracy is by increasing the amount of data collected. By repeating the experiment multiple times and taking an average of a large sample size of results you can minimise the effects that irregularities and outliers have on the final coefficient of restitution. The ideal sample size would be to have an infinite number of values to determine the coefficient from, which would effectively remove any uncertainty in the data collected, however this isn't feasible for an experiment being carried out one time in a three hour lab.

5 Conclusion:

We found the coefficient of restitution for Ball 1 to be:
0.75 ± 0.00741 using method one,
0.82 ± 0.02942 using method 2.

We found the coefficient of restitution for Ball 2 to be:
0.82 ± 0.00781 using method one,
0.83 ± 0.03685 using method 2.

6 Abstract:

The coefficient of two similar bouncy balls was found to be between (0.75 ± 0.00741 - 0.82 ± 0.02942) and (0.82 ± 0.00781 - 0.83 ± 0.03685) respectively, regardless of drop height.
These values were found using the relationship between drop height and initial bounce height as well as number of bounces n and the maximum height the ball reaches at that height h_n assuming air resistance is negligible. The entire concept and defining characteristic for a bouncy ball is its ability to forcibly rebound after colliding with a surface, the coefficient of restitution is extremely important as it allows us to assign a numerical value to an objects ability to rebound or 'bounce'. Our findings for the coefficient of restitution for a bouncy ball falls most closely in line with that of a golf ball or table tennis ball as reported by the Institute of Physics (IOP)[1]

References:

[1] A. Haron and K. A. Ismail. "Coefficient of restitution of sports balls: A normal drop test". IOP Conference Series: Materials Science and Engineering, 36:012038, sep 2012.

Derivation from Eq. 1 to Eq. 2

1.

$$e = \frac{v_2}{v_1}$$

Potential Energy E_P becomes Kinetic Energy E_K ($E_P = E_K$)

2.

$$mgh = \frac{1}{2}mv^2$$

Solving this equation for v:

3.

$$v = \sqrt{2gh} \text{ or } v = (2gh)^{\frac{1}{2}}$$

4.

$$v_1 = (2gh_1)^{\frac{1}{2}}$$
$$v_2 = (2gh_2)^{\frac{1}{2}}$$

Change in potential energy:

5.

$$\Delta E = mg(h_2 - h_1)$$

6.

$$e = \frac{v_2}{v_1} = (\frac{h_2}{h_1})^{\frac{1}{2}}$$