M1 and M2: Motion along a straight line
One dimensional uniform
and non-uniform motion

Jamie Lee Somers,
(DC171) B.Sc in Applied Physics.

Tuesday 8^{th} and Tuesday 15^{th} October, 2019
10:00 A.M - 1:00 P.M

1 Experimental Details:

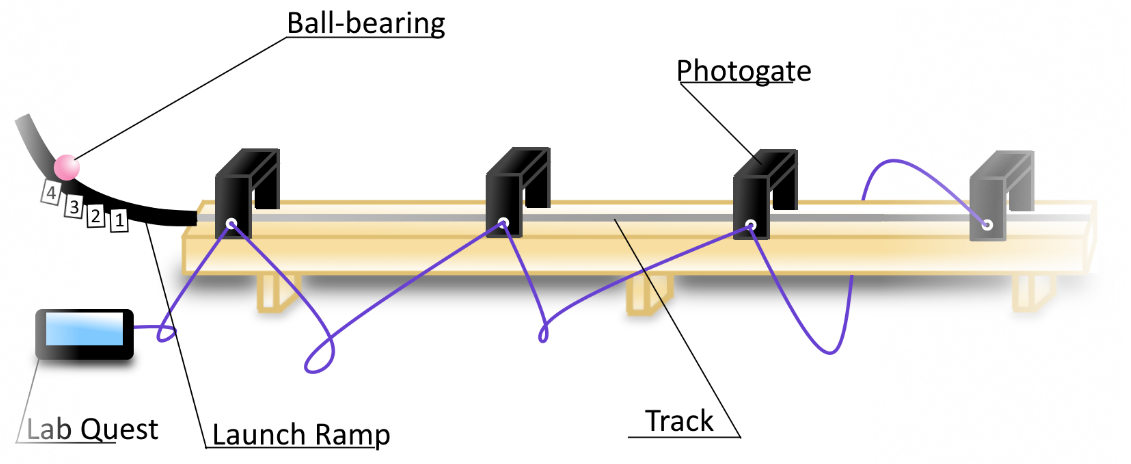

Figure 1.1: Diagram of Experimental Setup (M1)

The experimental setup of Fig. 1.1 is such that the ball can be released from different pre-determined heights and will travel in a straight line down the length of the track, triggering each photo-gate that it passes before coming to a rest at the end of the track. The distance between the photo-gates and the time it takes the ball to pass each one can be recorded as our experimental data.

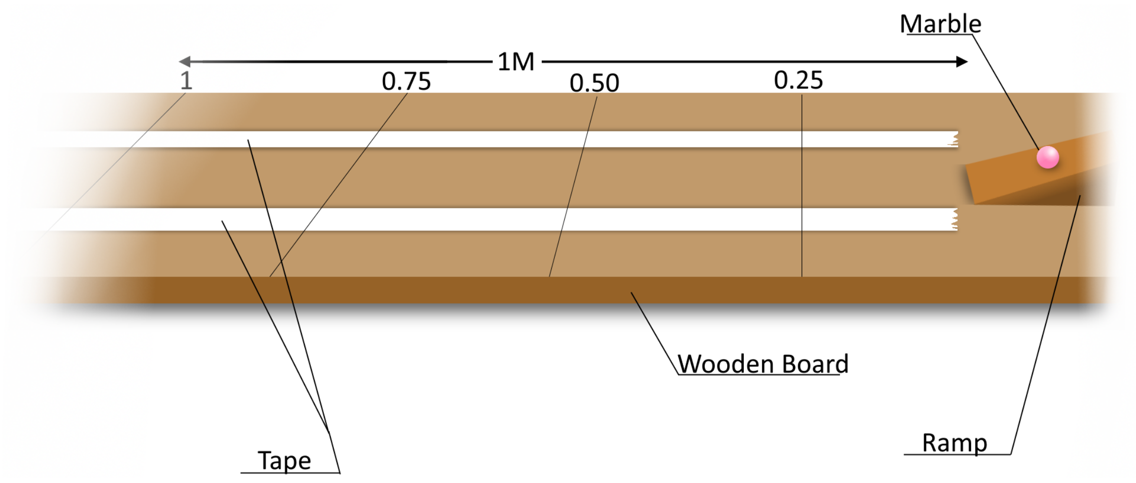

Figure 1.2: Diagram of Experimental Setup (M2)

The experimental setup of Fig. 1.2 involves the use of a ball rolling down a ramp along a wooden board which is taped with markings at 25cm intervals. The ball is believed to be accelerating down the length of the board, as it reaches each interval a stopwatch is used to measure the time it took the ball to reach each interval. Ideally we should observe that it takes less time for the ball to pass each interval as it accelerates. This is then repeated with the wooden board at an incline opposite the ramp to represent deceleration.

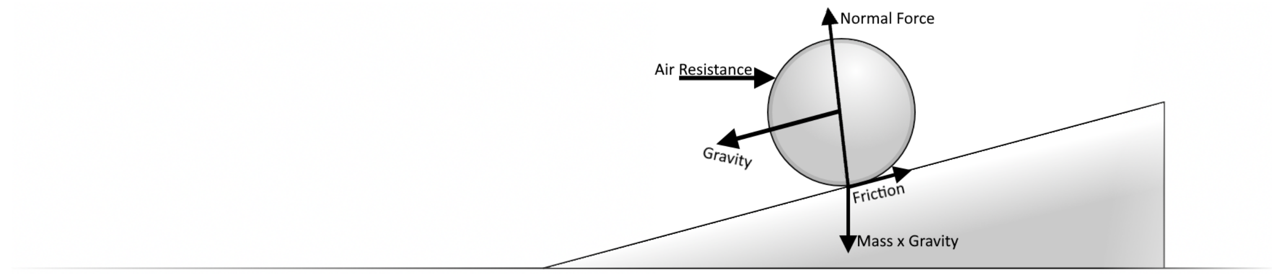

Figure 1.3: Forces acting on ball

We know from our understanding of physics that at all times there are forces acting on a body, and in this experiment our ball is no different, especially since it is undergoing motion.
Regardless of whether the ball is moving or not we can always assume that while it is on earth it is feeling the force of gravity pulling it downwards (Gravity), this force is amplified depending on how heavy the object is (Mass x Gravity), as we had the ball resting on a ramp there was an opposite force pushing it back up with an equal strength (Normal Force), which is why the ball was able to remain on the ramp and not get pulled straight through. If the ball was resting on a flat surface it would have remained stationary as all of these forces balanced with each other, however our ramp was such that it had a steep incline which resulted in gravity pushing it downwards in a direction with the least resistance. While this motion was taking place two forms of friction occurred, the friction between the ball and the surface of the ramp (Friction) as well as the air molecules hitting off the ball as it rolled downwards (Air Resistance).
All of these forces equated to the ball rolling in the fashion which was observed in the laboratory.

As we were using a metre stick to record the distance between our light gates, we can assume our position measurement errors are equal to the uncertainty of a metre stick, which is assumed to be $m \pm 1x10^{-3}$ m.
The timer attached to the light gate was able to measure our time component down to 5 decimal places with an uncertainty of $t \pm 1x10^{-5}$ s , since we had to leave our values at 3 decimal places as a result of the metre sticks uncertainty, it is extremely unlikely that our time measurements are inaccurate.

2 Results and Analysis:

2.1 Study of linear uniform motion

Table 1: Uniform motion Position: 1

Positions (m)	Times (s)
0	0
0.55	0.81
0.95	1.41
1.55	2.30

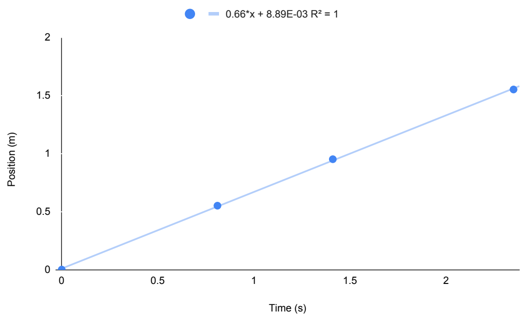

Figure 2.1: Graph of: Position 1 vs Time

Table 2: Uniform motion Position: 2

Positions (m)	Times (s)
0	0
0.45	0.61
0.85	1.13
1.35	1.90

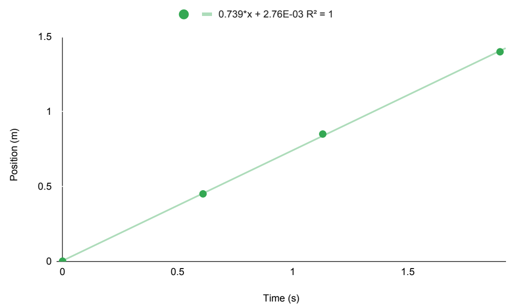

Figure 2.2: Graph of: Position 2 vs Time

Table 3: Uniform motion Position: 3

Positions (m)	Times (s)
0	0
0.40	0.45
0.95	1.08
1.35	1.56

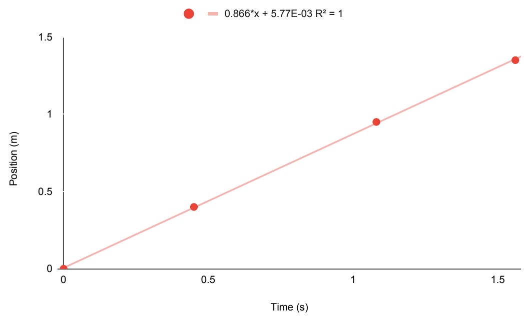

Figure 2.3: Graph of: Position 3 vs Time

Table 4: Uniform motion Position: 4

Positions (m)	Times (s)
0	0
0.50	0.55
0.90	0.97
1.30	1.43

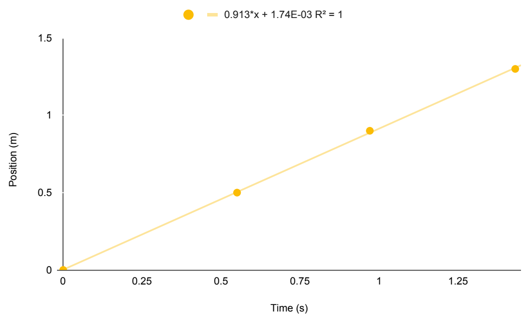

Figure 2.4: Graph of: Position 4 vs Time

Due to the experiment being based around linear motion in a straight line it would make sense that all of our graphs representing the position in relation to time would appear as straight lines.
Each of the tables contain data from the experiment with different distances between the light gates and the corresponding time it took the ball to reach those light gates however the graphs remain linear.

Using google sheets we can determine how accurate our data points hit the line of best fit by getting an R^2 Value, the closer our value is to 1, the better our data points hit our line of best fit. As we can see in Fig 3.1 to Fig 3.4 our R^2 value is consistently 1.

The big difference between our four graphs is the slope, as we go from Fig 3.1 through to Fig 3.4 we see that the value of our slope keeps increasing: {0.66, 0.739, 0.866 and 0.913}. This increase in slope is due to the velocity of the ball changing throughout the experiment, over the course of the 4 measurements the ball was dropped from greater and greater heights thus increasing its velocity.
Although our experiment was extremely accurate for the most part- there are still inaccuracies present; such as the y-intercept which identifies the origin position of the ball at $t = 0$, however this didn't perfectly map over as we would expect our Y-intercept to be zero.

2.2 Motion at constant velocity and conservation of mechanical energy

Table 5: Energy calculations to test conservation of mechanical energy

Starting height (m)	Velocity (m·s^{-1})	Kinetic energy (J)	Potential energy (J)	% difference
0.035	0.66	$1.74x10^{-3}$	$2.75x10^{-3}$	36.72
0.055	0.739	$2.18x10^{-3}$	$4.32x10^{-3}$	49.54
0.075	0.866	$3.00x10^{-3}$	$5.89x10^{-3}$	49.07
0.095	0.913	$3.33x10^{-3}$	$7.46x10^{-3}$	55.36

Using the equation:

$$E_{\mathrm{k}} = 0.5\,x\,mv^2 \tag{1}$$

We can workout the Kinetic Energy since we know the mass of the ball is 0.008kg

$$1.74x10^{-3} = 0.5\,x\,(0.008)(0.66)^2$$
$$2.18x10^{-3} = 0.5\,x\,(0.008)(0.739)^2$$
$$3.00x10^{-3} = 0.5\,x\,(0.008)(0.866)^2$$
$$3.33x10^{-3} = 0.5\,x\,(0.008)(0.913)^2$$

We can work out the potential energy using the equation:

$$E_{\mathrm{p}} = mgh \tag{2}$$

$$2.75x10^{-3} = (0.008)(9.81)(0.035)$$
$$4.32x10^{-3} = (0.008)(9.81)(0.055)$$
$$5.89x10^{-3} = (0.008)(9.81)(0.075)$$
$$7.46x10^{-3} = (0.008)(9.81)(0.095)$$

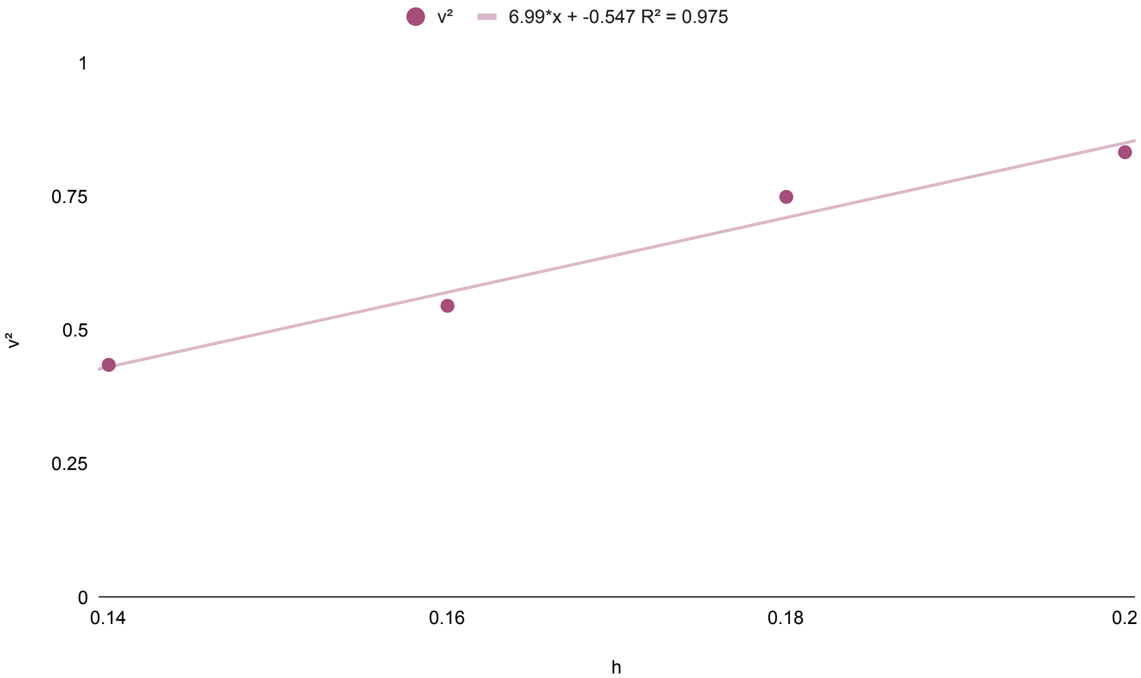

Figure 2.5: graph of v^2 vs h

Although Potential energy accounts for all of the energy that could possibly be given off during the rolling of the ball down a ramp, not all of this energy is given off in the form of kinetic energy.

We know from carrying out the experiment that some of the energy generated by rolling a marble from the top of a ramp is given off in the form of sound energy, as we can audibly hear the sound energy as it is being produced,

we can also assume that due to the friction of the marble against the surface of the wooden board that some small fraction of the potential energy was given off in the form of heat energy.

From our graph we were able to determine the value for g as 6.99 ms^{-2}, this is different from the expected value of 9.81 ms^{-2}. Later in this report we will compare these two values.

2.3 Study of non-uniform motion

Table 6: Recorded Stopwatch times at each interval (Acceleration)

Distance	$Time_1$	$Time_2$	$Time_3$	$Time_4$	$Time_5$	$Time_6$	$Time_7$	$Time_8$	$Time_9$	$Time_{10}$
0.25m	0.43	0.43	0.41	0.94	0.94	0.93	0.49	0.42	0.67	0.35
0.5m	0.73	0.82	0.80	1.41	1.41	1.32	0.89	0.82	0.91	0.67
0.6m	0.88	0.97	0.95	1.53	1.56	1.47	1.04	0.97	1.06	0.82
0.65m	0.93	1.02	1	1.61	1.61	1.52	1.09	1.02	1.11	0.87
0.75m	0.98	1.04	1.04	1.71	1.68	1.58	1.12	1.05	1.13	0.96
0.80m	1.1	1.16	1.16	1.83	1.8	1.7	1.24	1.17	1.25	1.08
0.90m	1.13	1.19	1.18	1.86	1.83	1.73	1.27	1.2	1.28	1.11
1m	1.14	1.25	1.19	1.86	1.93	1.73	1.27	1.22	1.29	1.22

Table 7: Average Stopwatch times at each interval (Acceleration)

Distance (m)	$Time_{Avg}$ (s)
0.25	0.6
0.5	0.98
0.60	1.13
0.65	1.18
0.75	1.23
0.80	1.35
0.90	1.38
1	1.41

We can then represent our data in Table 7 on a graph:

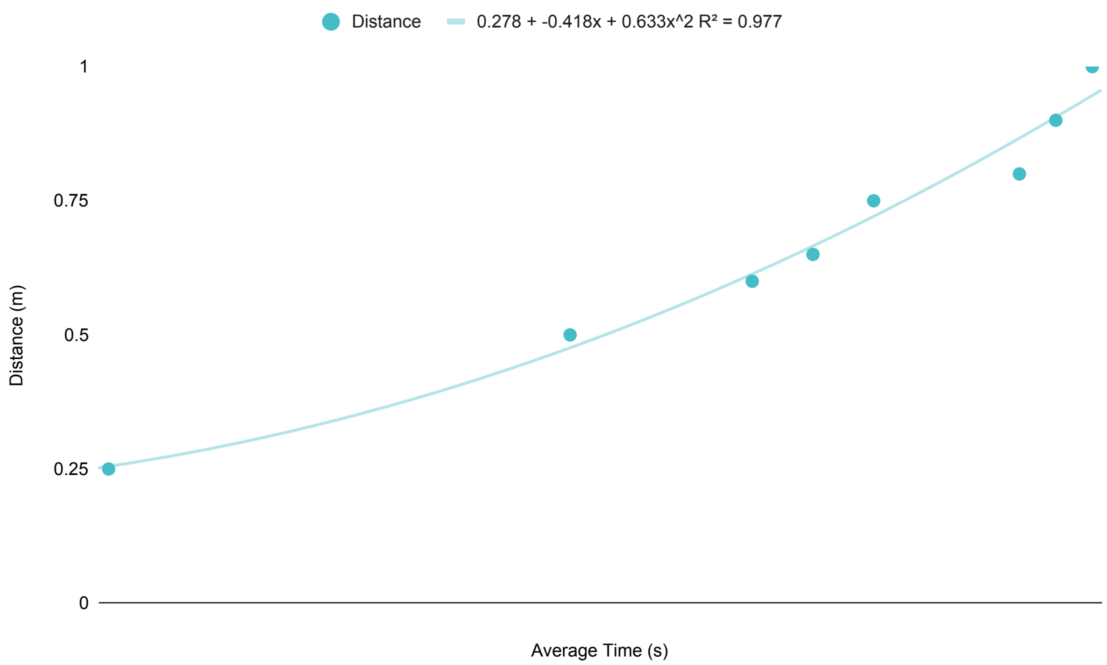

Figure 2.6: Average Time vs Distance (Acceleration)

Table 8: Recorded Stopwatch times at each interval (Deceleration)

Distance	Time_1	Time_2	Time_3	Time_4	Time_5	Time_6	Time_7	Time_8	Time_9	Time_{10}
0.25m	0.27	0.31	0.27	0.23	0.25	0.28	0.28	0.28	0.23	0.32
0.5m	0.61	0.68	0.62	0.66	0.72	0.70	0.58	0.67	0.60	0.74
0.6m	0.81	0.88	0.82	0.86	0.92	0.90	0.78	0.87	0.80	0.94
0.65m	0.93	1.00	0.94	0.98	1.04	1.02	0.90	0.99	0.92	1.06
0.75m	1.09	1.14	1.19	1.17	1.19	1.18	1.05	1.15	1.07	1.25
0.80m	1.31	1.36	1.41	1.39	1.41	1.40	1.27	1.37	1.29	1.47
0.90m	1.44	1.49	1.54	1.52	1.54	1.53	1.40	1.50	1.42	1.60
1m	1.74	1.78	1.83	1.80	1.94	1.73	1.62	1.78	1.60	1.85

Table 9: Average Stopwatch times at each interval (Deceleration)

Distance (m)	Time_{Avg} (s)
0.25	0.27
0.5	0.66
0.60	0.86
0.65	0.98
0.75	1.15
0.80	1.37
0.90	1.50
1	1.78

We can then represent our data in Table 10 on a graph:

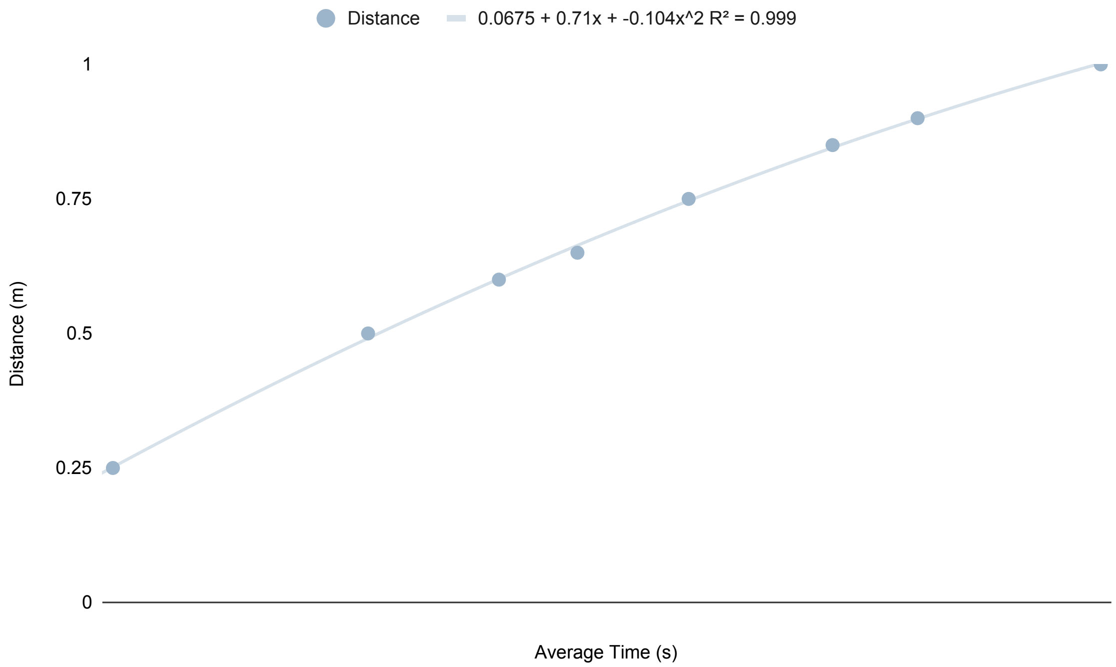

Figure 2.7: Average Time vs Distance (Deceleration)

Now that we have the equation of the line for both accelerating and decelerating motion, we can compare the coefficients to the equation of motion that the ball follows and we can extract information about the acceleration a_{x} and initial velocity v_0 of the ball. The equation that corresponds to our balls motion is:

$$\frac{1}{2}\,a_{\mathrm{x}}\,t^2 + v_{0,\mathrm{x}}\,t + x_0 \tag{3}$$

Which when compared to our two equations of the line:

$$\frac{0.633}{2}\,x^2 - 0.418\,x + 0.278$$
$$\frac{-0.104}{2}\,x^2 + 0.71\,x + 0.0675$$

Tells us that our value for $a_{\mathrm{x}} = 1.266ms^{-2}$ *and* $-0.208ms^{-2}$ respectively, and our values for $v_{0,\mathrm{x}} = -0.418ms^{-1}$ *and* $0.71ms^{-1}$ respectively.

We can now get the derivation of our motion equations and graph this new equation with our previous values. Our equation of motion for acceleration was as follows:

$$f(x) = 0.633\,x^2 - 0.418\,x + 0.278 \tag{4}$$

Therefore our derivative of f(x) with respect to x would be as follows:

$$f'(x) = 1.266\,x - 0.418 \tag{5}$$

Graphing our new function using our t values as x we get Fig. 2.8:

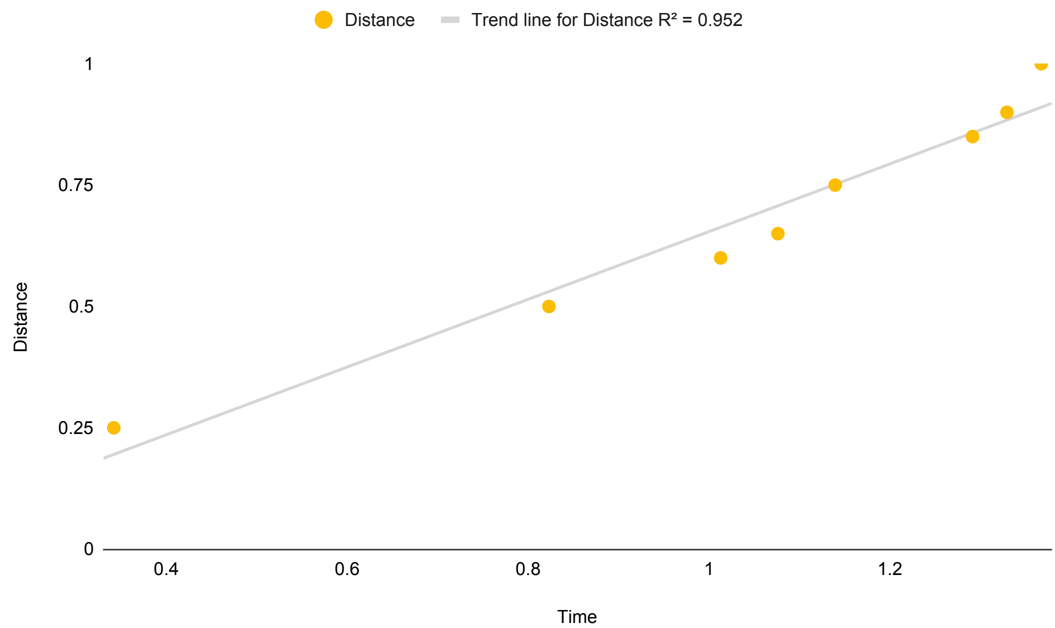

Figure 2.8: Derivation of motion equation (Acceleration)

Our equation of motion for deceleration was as follows:

$$f(x) = -0.104\,x^2 + 0.71\,x + 0.0675 \tag{6}$$

Therefore our derivative of f(x) with respect to x would be as follows:

$$f'(x) = -0.208\,x + 0.71 \tag{7}$$

Graphing our new function using our t values as x we get Fig. 2.9:

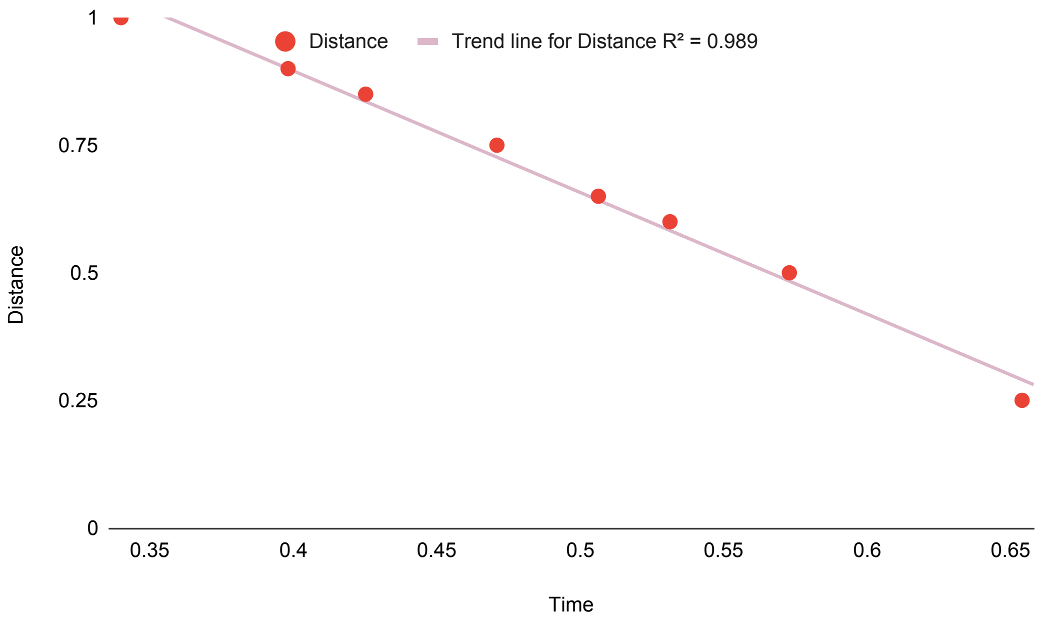

Figure 2.9: Derivation of motion equation (Deceleration)

We can also calculate an ideal, frictionless acceleration value a using the equation:

$$\pm g\, sin\, \alpha = a \tag{8}$$

Where α is equal to the angle our wooden board was tilted at:
$(\pm 9.81)(sin(2.44)) = \pm 0.418$

Although this value is similar to our deceleration value (-0.418 ms^{-2} vs -0.208 ms^{-2}), The value is extremely dissimilar from our acceleration value (0.418 ms^{-2} vs 1.266 ms^{-2}).

This could represent that there was something wrong with how we carried out our acceleration measurements that wasn't present when measuring our deceleration values.

3 Discussion

3.1 Friction

The biggest inaccuracy within our experiment is the assumption that no energy is lost due to the contact the ball is making with the surface it is rolling on, if we were able to completely remove friction from our experiment it would most likely result in our accuracy increasing.

Although every effort was made to ensure that there was nothing located on the ramp and track that would interfere with the balls motion unfortunately any microscopic irregularities in the ball or surface can play a roll in altering the balls motion.

In order to improve our accuracy and reduce friction we would need to incorporate a method of smoothing down the surface of the ball and track, or reducing the contact between the two. Two options could be to sand down any wooden surfaces before carrying out the experiment or introducing a lubricant which can reduce the contact made between the two surfaces, however both of these methods bring other sources of errors which could affect the results such as the sanding altering the heigh measurements of the track and the lubricant affecting the velocity of the ball.

3.2 Values for gravity

Throughout our analysis of the experiment we were unable to determine a consistent value for gravity, however we were able to work out some values that roughly represent the known value of 9.81 ms^{-2}. Using our v^2 vs h graph (Fig. 2.5) we were able to obtain a value of 6.99 ms^{-2}. Which we can compare to our known value using the formula:

$$\frac{Known\,Value - Obtained\,Value}{Known\,Value}\ x\ 100 \tag{9}$$

$\frac{9.81-6.99}{9.81}\ x\ 100 = 28.75\%$

Therefore our 6.99 ms^{-2} result is fairly close to the value we expected to obtain from carrying out this experiment.

3.3 Differences between uniform and non-uniform motion

Although both experiments involved a ball rolling along a straight line across a surface and visually looked almost identical, as we can see from our results, there is quite a big difference between what happens to a ball in uniform motion vs non uniform motion.

The most striking difference being the variability in speed, in M1 our ball was moving with a constant acceleration and therefore our calculations were linear and straightforward, however when we carried our M2 our ball had a variable speed due to a change in acceleration/deceleration and therefore made our calculations more complex in nature.

4 Appendix: Pre-lab Problem

Table 10: Position-time data for the three balls in same reference frame

Time(s)	Position Ball 1 (m)	Position Ball 2 (m)	Position Ball 3 (m)
0	0.4	0.2	2
0.5	0.5	0.36	1.8
1	0.6	0.52	1.6
1.5	0.7	0.68	1.4
2	0.8	0.84	1.2
2.5	0.9	1	1
3	1	1.16	0.8
3.5	1.1	1.32	0.6
4	1.2	1.48	0.4
4.5	1.3	1.64	0.2
5	1.4	1.8	0

We can graph this relationship out using Google Sheets:

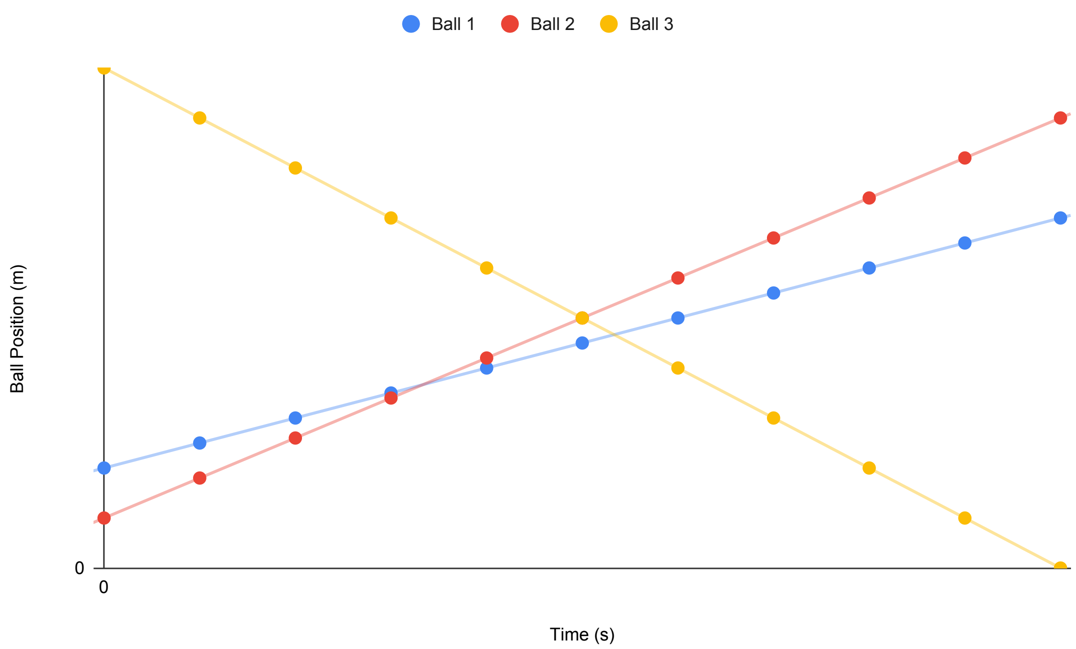

Figure 4.1: Graph of Position vs Time for three separate balls

We can calculate the velocity of each ball using the equation:

$$Velocity = \frac{Distance}{Time} \tag{10}$$

$$Ball\,1 : \frac{1.4-0.4}{5-0} = 0.2\,m\,s^{-1}$$

$$Ball\,2 : \frac{1.8-0.2}{5-0} = 0.32\,m\,s^{-1}$$

$$Ball\,3 : \frac{0-2}{5-0} = -0.4\,m\,s^{-1}$$

Since we know x and y co-ordinates for each of the balls we can work out the mathematical equation of each line using the formula:

$$\frac{y_2-y_1}{x_2-x_1} = m \tag{11}$$

To find the slope and equation:

$$y - y_1 = m(x - x_1) \tag{12}$$

To find the equation of the line

$$\frac{1.4-0.6}{5-1} = 0.2$$
$$y - 0.6 = 0.2(x-1)$$
$$\mathbf{Ball\,1 : y = 0.2x + 0.4}$$

$$\frac{1.8-0.52}{5-1} = 0.32$$
$$y - 0.52 = 0.32(x-1)$$
$$\mathbf{Ball\,2 : y = 0.32x + 0.2}$$

$$\frac{0-1.6}{5-1} = -0.4$$
$$y - 1.6 = -0.4(x-1)$$
$$\mathbf{Ball\,3 : y = -0.4x + 2}$$

In order to work out mathematically at what point Ball 1 and Ball 3 are at the same position we must make the equation of Ball 1 and Equation of Ball 3 equal to each other:

$$0.2x + 0.4 = -0.4x + 2$$

And we end up with the equation

$$0.6x = 1.6$$
$$\mathbf{x = 2.\dot{6}\,s}$$
$$\mathbf{y = 0.9\dot{3}\,m}$$

We can work out the position of ball number 2 by subbing in our x co-ordinate and seeing what value we get for y:

$$\mathbf{Ball\,2 : y = 0.32(2.\dot{6}) + 0.2}$$

$$\mathbf{Ball\,2 : y = 1.05\dot{3}m}$$

It takes 5 seconds for Ball 1 to travel 1 metre of the track.

It takes 5 seconds for Ball 2 to travel 1.6 metres down the track.

It takes 5 seconds for Ball 3 to travel the full 2 metre track

Ball 3 reaches the opposite end of the track first while Ball 1 is 1.4 metres down the track and Ball 2 is 1.8 metres down the track. 5.625 seconds for Ball 2 to finish 10 seconds for Ball 1 to finish