Investigation of Elasticity

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1 Report and Data Analysis:

Table 1: Experiment 1 Rubber Band

Mass / kg	Length / m	Extension / m
0	0.12	0
0.01	0.122	0.002
0.02	0.124	0.004
0.03	0.126	0.006
0.04	0.128	0.008
0.05	0.129	0.009
0.06	0.131	0.011
0.07	0.133	0.013
0.08	0.135	0.015
0.09	0.136	0.016
0.1	0.138	0.018

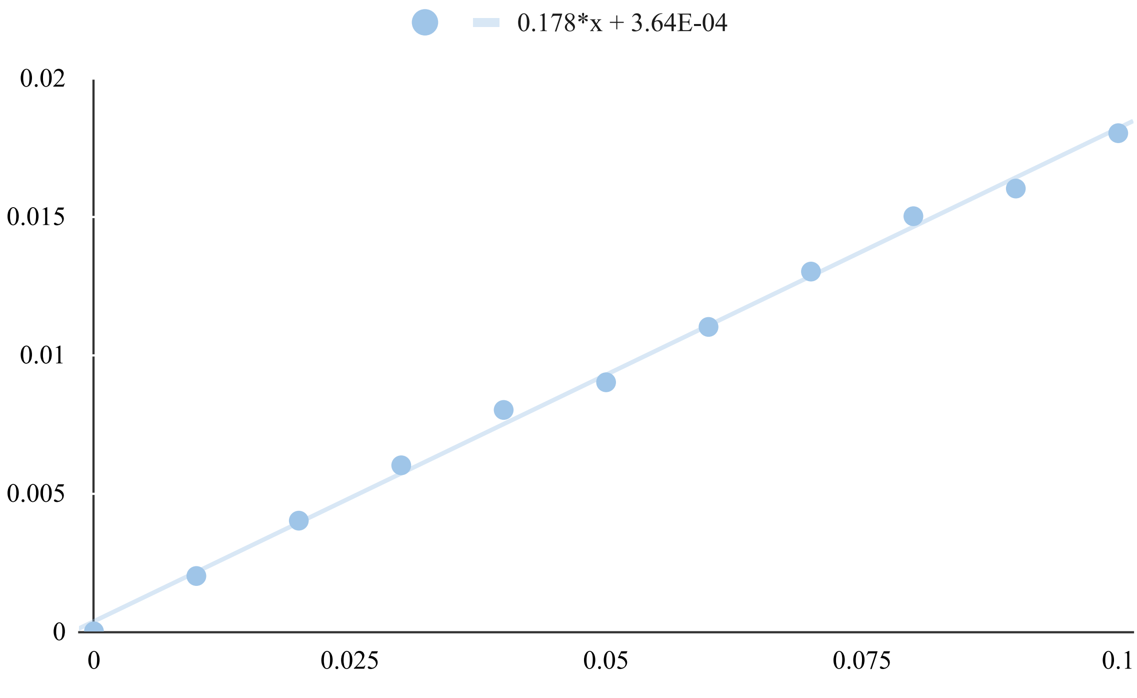

Figure 1.1: Mass (kg) vs Extension (m)

Table 2: Experiment 1 Steel Spring

Mass / kg	Length / m	Extension / m
0	0.114	0
0.01	0.1145	0.0005
0.02	0.1145	0.0005
0.03	0.115	0.001
0.04	0.115	0.001
0.05	0.1155	0.0015
0.06	0.1155	0.0015
0.07	0.1165	0.0025
0.08	0.119	0.005
0.09	0.121	0.007
0.1	0.123	0.009

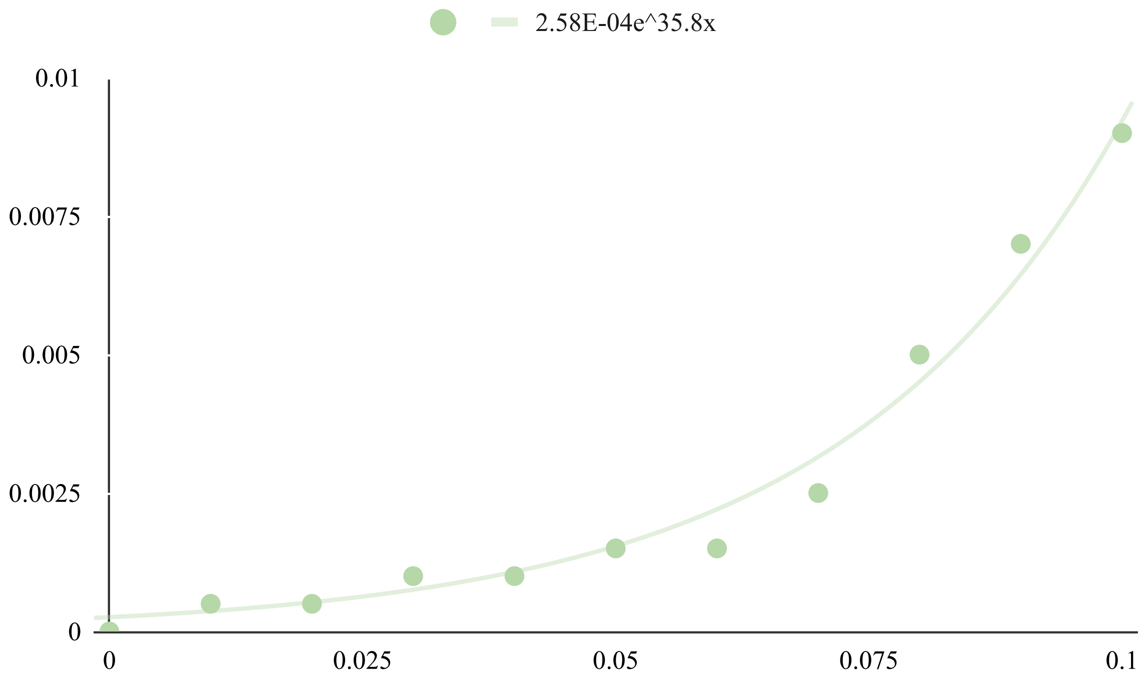

Figure 1.2: Mass (kg) vs Extension (m)

We can determine that the slope of our first graph is 0.178, and that the corresponding k values can be calculated using Hooks Law:

$$F = -k\Delta l \tag{1}$$

We already have a value for Δl, and we can determine a value F_{s} using:

$$W = -F_{\mathrm{s}} \tag{2}$$

And we can calculate W using:

$$W = mg \tag{3}$$

This implies that:

$$mg = k\Delta l \tag{4}$$

So therefore we can say that

$$\Delta l = \frac{g}{k} m \tag{5}$$

We can manipulate the final equation to give us:

$$\frac{gm}{\Delta l} = k$$

Where g = 9.81, m = our final mass, Δl = The change in length.

For the Rubber Band we get:

$$\frac{(9.81)(0.1)}{0.138-0.12} = 54.5$$

For the Metal Spring we get:

$$\frac{(9.81)(0.1)}{0.123-0.114} = 109$$

(3) Error calculations

We can calculate the error associated with our values by using the equation:

$$Y = AB \; or \; Y = \frac{A}{B} \rightarrow \frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \frac{\Delta B}{B} \tag{6}$$

Where ΔA = the error associated with the mass, and ΔB = the error associated with the change in length in our equation.

Our Δl measurement was calculated by subtracting our original length from our final length, both of which had an uncertainty of ± 0.001 m, we therefore add the uncertainty of the two measurements we used, to get a total uncertainty of Δl of ± 0.002 m.

All other variables in the calculation seemingly have no uncertainty associated with them, we assume that g is exactly 9.81 ms^{-2} and the mass of the weights used had no uncertainty associated with the printed value.

Rubber Band Uncertainty:

$$\frac{0.002}{0.018} + \frac{0}{0.1} = \frac{1}{9}$$

$$\frac{1}{9} \div 54.5 = 0.002$$

Steel Spring Uncertainty:

$$\frac{0.002}{0.009} + \frac{0}{0.1} = \frac{2}{9}$$

$$\frac{2}{9} \div 109 = 0.002$$

Therefore our k values including their uncertainties are as follows:

Rubber Band, Spring Constant: 54.5 ± 0.002

Steel Spring, Spring Constant: 109 ± 0.002

Hook's Law (Eq. 1) tells us that the force a spring exerts is proportional to the displacement of the spring, This means that a spring with a large displacements have a small spring constant, and springs that are stiff with low displacement have a high spring constant. This is exactly what we observe as our Rubber spring has twice the displacement of the Steel Spring (0.018 m to 0.009 m) and its spring constant is exactly half that of the steel spring (54.5 to 109).

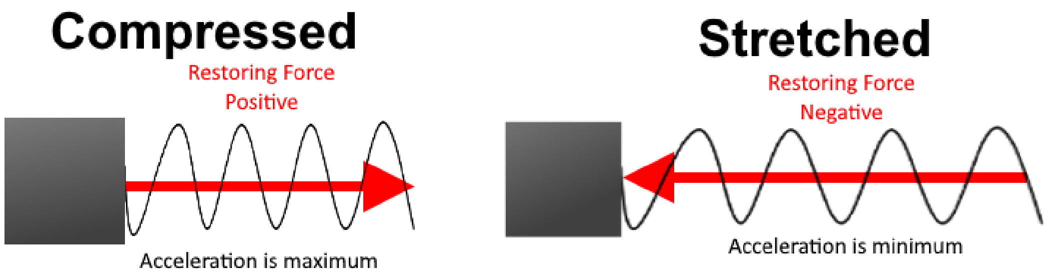

Figure 1.3: Oscillating Spring Mass System

Table 3: Experiment 2: Steel Spring

Mass / kg	Time / s (30 oscillations)	Period / s	Period Squared / s^2
0.1	11.08	0.3693	0.1364
0.15	12.06	0.402	0.1616
0.2	12.99	0.433	0.1875
0.25	14.93	0.4976	0.2476
0.3	15.63	0.521	0.2714
0.35	16.84	0.5613	0.3151
0.4	18.15	0.605	0.3660
0.45	19.26	0.642	0.4122
0.5	20.11	0.6703	0.4493
0.55	21.06	0.702	0.4928

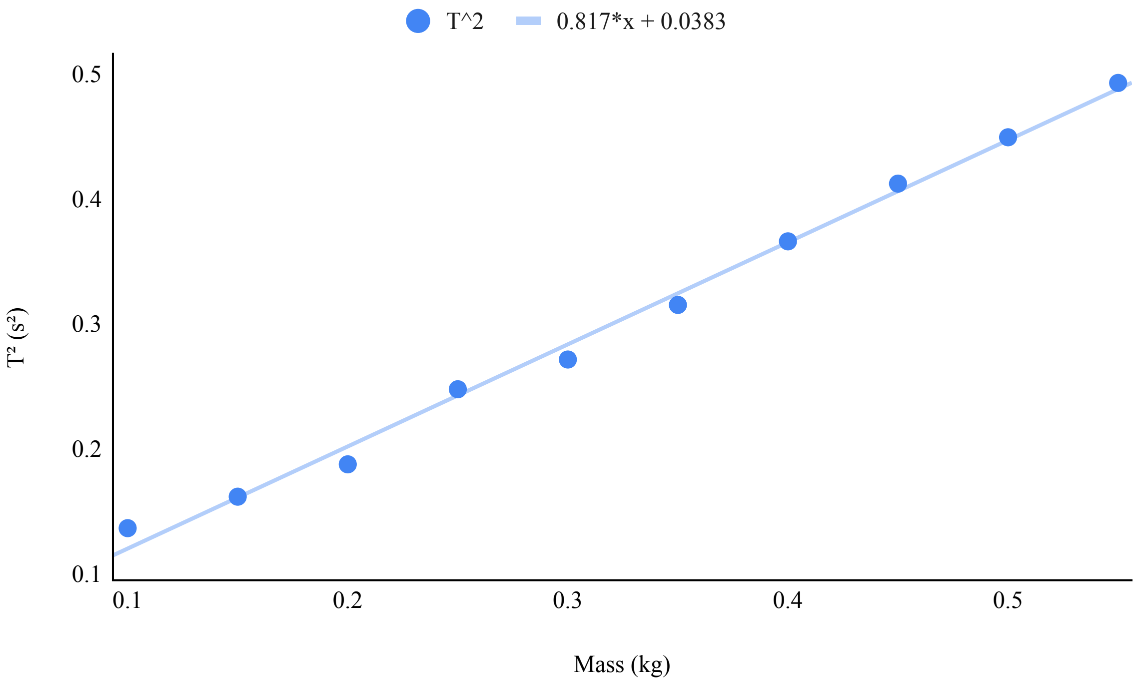

Figure 1.4: Mass (kg) vs $\mathrm{T}^2(s^2)$

We know from simple harmonic motion that our slope should be equivalent to: $\frac{4\pi^2}{k}$

$$\frac{4\pi^2}{109} = 0.362$$

Our y-intercept should be comparable to: $\frac{4\pi^2 m_{\mathrm{S}}}{k}$

$$\frac{4\pi^2(0.1)}{109} = 0.036$$

Our value for the slope doesn't line up with the expected value, however our y-intercept does closely relate to the expected.
Using the value we got for the slope from our Fig. 1.3: 0.817, we can work out what the spring constant of the steel spring was supposed to be.

$$\frac{4\pi^2}{m} = k$$

$$\frac{4\pi^2}{0.817} = 48.3$$

Using the value we got for the y-intercept from Fig. 1.3: 0.0383, we can work out what the effective mass of the spring.

$$\frac{(k)(y{-}intercept)}{4\pi^2} = m_{\mathrm{s}}$$

$$\frac{(109)(0.0383)}{4/pi^2} = 0.106$$

This effective mass value is extremely similar to the mass used in our experiment. 0.106kg to 0.1kg. Despite us assuming that the steel spring had an effective mass of 0.1kg, there may have been impurities in the spring which resulted in an add weight of 0.006kg

Using the LINEST function we can determine the uncertainty associated with the calculated slope and y-intercept, using this function we get:

$$\text{Slope: } 0.362 \pm 0.003$$

$$\text{Y-Intercept: } 0.036 \pm 0.01$$

Since our slope was used to calculate the spring constant and all the other values were given, that means this uncertainty is also the uncertainty of the spring constant:

$$\text{Spring Constant: } 48.3 \pm 0.003$$

We used both the Spring Constant and the Y-Intercept to find the effective mass of the string, so therefore the combined uncertainty is:

$$\text{Effective Mass: } 0.106 \pm 0.013$$

2 Conclusion:

We determined that the spring constant of a rubber band was 54.5 ± 0.002, we used two methods to determine that the spring constant of a steel spring was either 109 ± 0.002 or 48.3 ± 0.003, However these two values are not within the experimental uncertainty margin of each other.