Focal Length & Lens-Makers Equation

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1 Introduction:

The main aims of this experiment was to discover a method of finding the focal length of a lens using easy to obtain experimental results and a mathematical formula, as well as using the lens-makers equation to work out the refractive index of the lenses.

Refraction is the phenomenon which occurs when light passes through a material that is non-opaque causing its speed to be altered, refraction is greater in non-opaque materials of high density. This change in speed results in the ray of light leaving the medium appearing bent.

Snell's Law is a formula which describes the relationship between the angle of light going into a material and the angle of light leaving the substance with respect to the refractive index of both materials. As such, the equation can be extremely helping in finding either: the angle a beam of light will take or the refractive index of a material, as long as the other variable is known.

In this experiment, two lenses were utilised, the goal was to focus an image of a filament bulb onto a screen which can be placed at variable distances away, the two lenses were:
A Convex Lens - Which was the primary lens used throughout the experiment.
A Concave Lens - which was placed in tandem with the convex lens later in the experiment.

The two main categories of lenses are Concave and Convex, these lens variants are dependent on the direction in which the lens curves, an outward curving lens is referred to as a Convex Lens, while an inward curving lens is labelled as a Concave Lens. Despite the categorization of these lenses, both are governed by the same equation:

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f} \tag{1}$$

Where d_0 = distance between object and lens, d_i = distance between image and lens, and f = the focal length of the lens.

Another method of finding the focal length of a lens, known as 'The Lens-Makers Equation':

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \tag{2}$$

Where n = refractive index of the lens, $R_1/_2$ = radii of curvature for the surface of the lens, both sides 1 and 2.

Despite both variants of the lens using this equation, the difference in lenses are noticeable due to the fact that in a convex lens R_2 is negative, as apposed to a concave lens where R_1 would be negative.

In this experiment we will actually be using 'The Lens-Makers Equation' to find 'n' the refractive index of the lens.

2 Methods:

Measuring Focal Length of Convex / Concave Lens

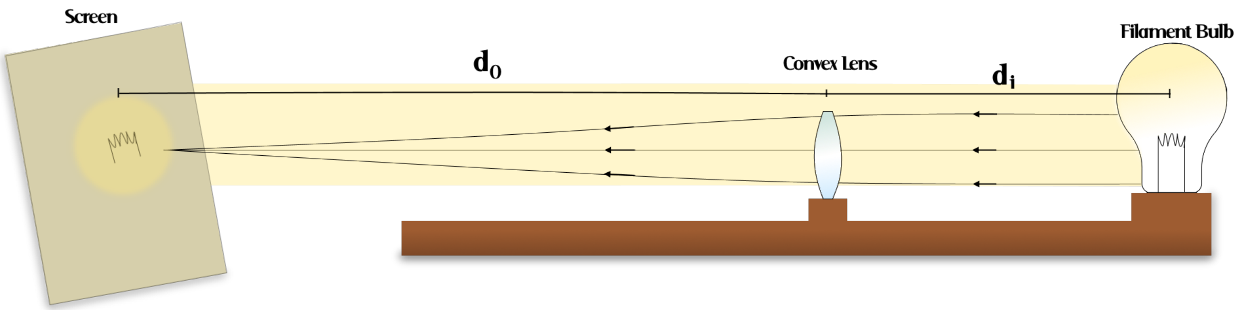

Figure 2.1: Experimental Set-up Diagram

The experimental set-up consists of a light box which houses a filament bulb (which doubles as our object),a concave and convex lens, a lens holder capable of housing our concave, convex lenses individually and combined, a screen which the image of our object can be projected onto, and a metre stick which can measure the distances required for our experimental data.

First a rough estimate of the focal length was found. We did this by placing the lens horizontal to the ground and trying to form a sharp image of the florescent lights from the ceiling onto the ground, once this was done the distance between the image and the lens was measured and that was assumed to be the rough focal length of the lens.

The lens was placed into the lens holder, the lens holders distance from the bulb was set to be greater than the rough estimate previously gotten otherwise an image does not form on the screen. Assume the apparatus is set up as shown above and the lens is placed outside the rough focal length an image of a bulb should appear on the screen.

The screen was moved in an attempt to increase the sharpness of the image displayed. Once this was achieved the measurements object distance and image distance were recorded. These distances were changed again attempting to get a sharp image on the screen and once again the measurements were recorded.

The focal length was found by plotting $\frac{1}{d_0}$ vs $\frac{1}{d_\mathrm{i}}$ and finding the intercept, as well as the uncertainty associated with it. As well as plotting $(d_0\, d_\mathrm{i})$ vs $(d_0 + d_\mathrm{i})$ where f is the slope, and also finding its uncertainty.

The concave lens was then added in combination with the convex lens and all of the steps were repeated with this new combination of lenses.

Focal Length of Lens using Radii of curvature

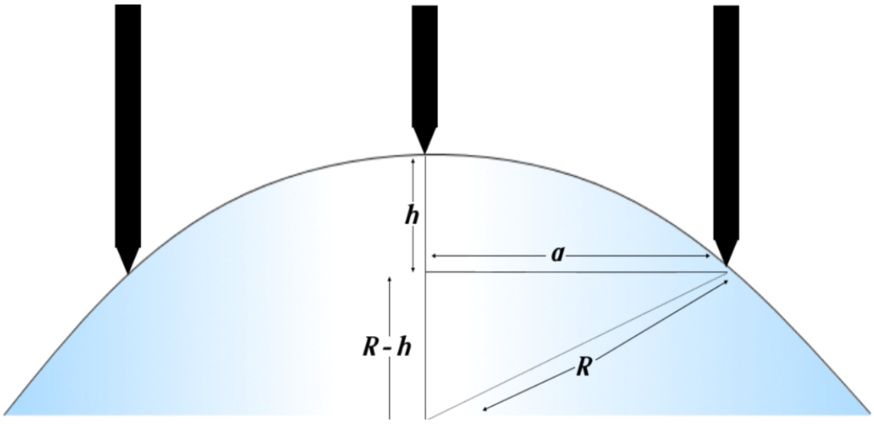

Figure 2.2: Experimental Set-up Diagram

The Spherometer was placed on a flat reflective surface in order to calibrate it, after which the distance from the central leg to an outer leg was measured.

The Spherometer was placed on the surface of the convex lens, and the number of rotations were counted until all four legs were resting on the lens evenly keeping in mind that 2 full rotations was 1 mm. h was measured and R was calculated.

Finally the focal length of the lens was found using Eq. 2 which was mentioned during the introduction, along with the associated error.

3 Results and Analysis:

Rough Focal Length (Convex): 20 cm
Rough Focal Length (Convex + Concave): 59 cm
Rough Focal Length (Concave): − 30 cm

Note: The 'Rough Focal Length' of the concave lens was calculated using the equation:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \tag{3}$$

Where F = combined focal length, f_1 = focal length of convex lens and f_2 = focal length of concave lens.

Therefore:

$$\frac{1}{59} - \frac{1}{20} = -\frac{1}{30}$$

Table 1: Focal Length Measurements (Convex)

d_0 (cm)	d_i (cm)	$\frac{1}{d_0}$	$\frac{1}{d_i}$	$d_0 + d_i$	$d_0\,d_i$
25	106.6	0.04	0.009	131.6	2,665
35	46.3	0.029	0.022	81.3	1,620.5
45	35.8	0.022	0.028	80.8	1,611
55	31.5	0.018	0.032	86.5	1,732.5
65	29.2	0.015	0.034	94.2	1,898
75	27.4	0.013	0.036	102.4	2,055
85	26.5	0.012	0.038	111.5	2,252.5
95	25.7	0.011	0.039	120.7	2,441.5

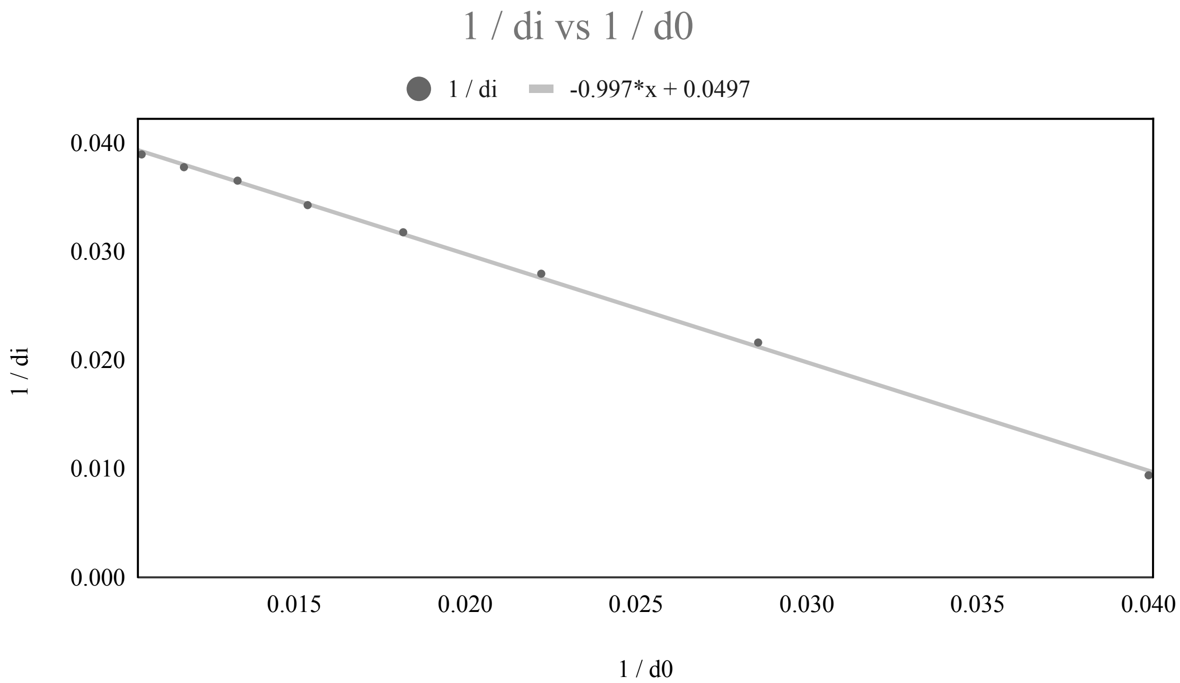

Figure 3.1: $\frac{1}{d_0}$ vs $\frac{1}{d_i}$

In this case, the y-intercept of our graph is: $\frac{1}{f}$

Focal Length:

$$0.0497^{-1} = 20.1207$$

The LINEST function tells us the uncertainty of the y-intercept is: $\pm$ 0.0004

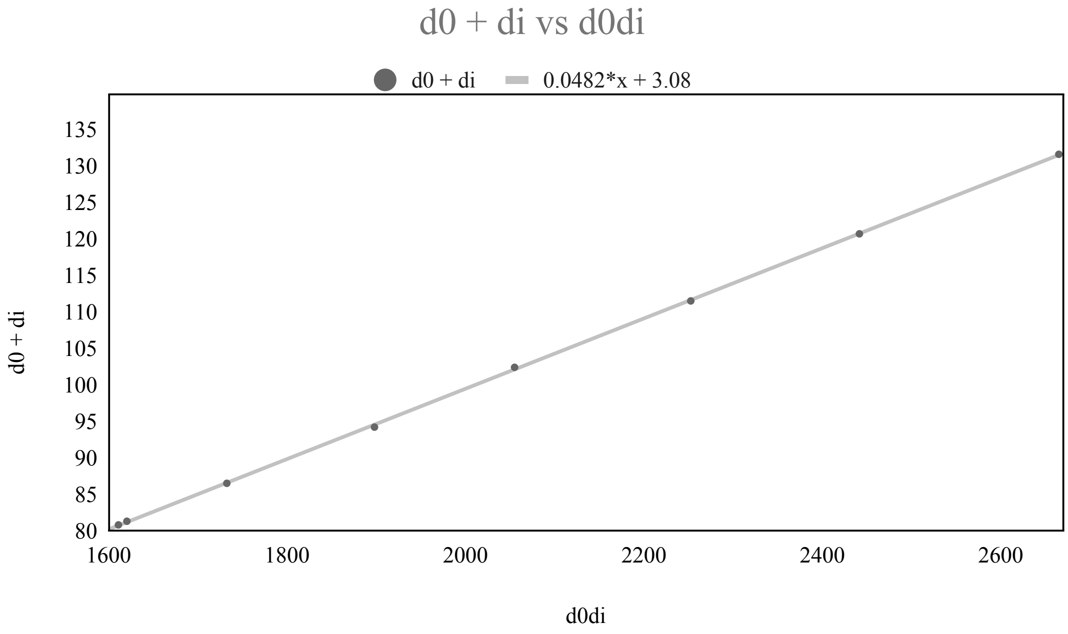

Figure 3.2: $d_0 + d_\mathrm{i}$ vs $d_0 d_\mathrm{i}$

In this case, the slope of our graph is: $\frac{1}{f}$

Focal Length:

$$0.0482^{\text{-1}} = 20.7469$$

The LINEST function tells us the uncertainty of the y-intercept is: $\pm$ 0.0862

Comparing Focal Lengths

- Method 1: 20.1207 cm ± 0.0004
- Method 2: 20.7469 cm ± 0.0862

Both of our methods gave us a focal length similar to that of our rough estimate of 20cm, however the first method gave us a closer value with a much lower uncertainty.

Table 2: Focal Length Measurements (Convex + Concave)

d_0 (cm)	d_i (cm)	$\frac{1}{d_0}$	$\frac{1}{d_i}$	$d_0 + d_i$	$d_0\ d_i$
65	229.5	0.0154	0.0044	294.5	14,917.5
70	199.5	0.0143	0.005	269.5	13,965
75	137.9	0.013	0.0073	212.9	10,342.5
80	106.7	0.0125	0.0094	186.7	8,536
85	169.4	0.0118	0.0059	254.4	14,399
90	153	0.01	0.0065	243	13,770
95	143.2	0.0105	0.007	238.2	13,604

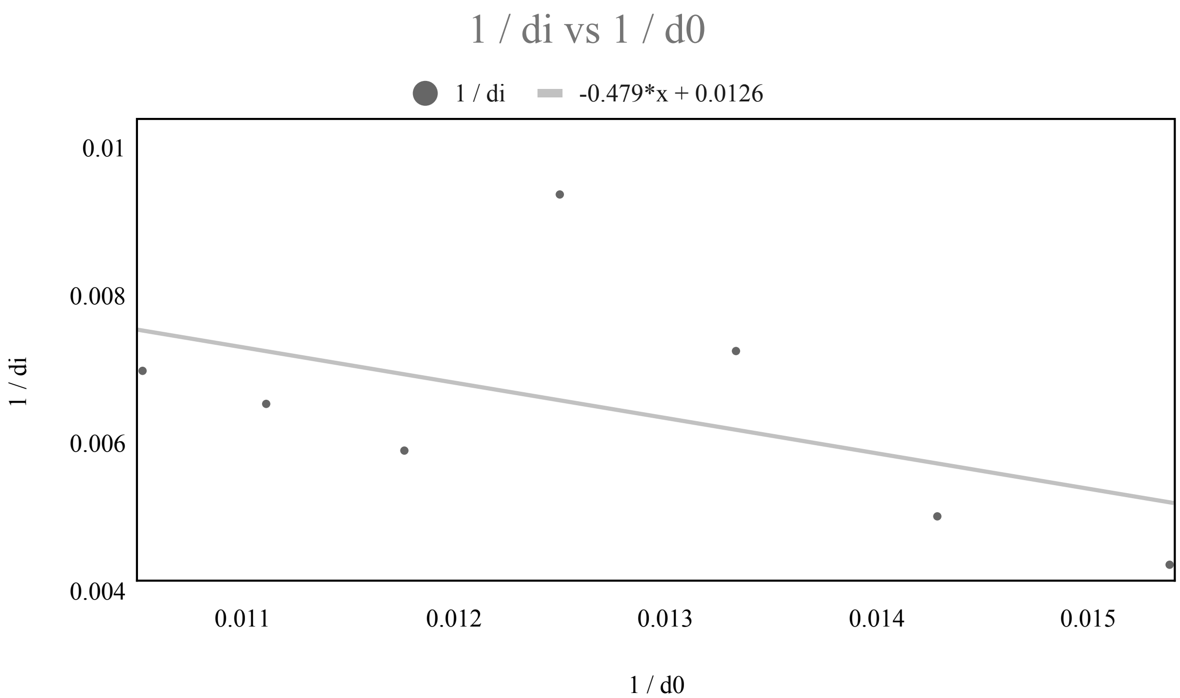

Figure 3.3: $\frac{1}{d_0}$ vs $\frac{1}{d_i}$

In this case, the y-intercept of our graph is: $\frac{1}{f}$

Focal Length:

$$0.0126^{-1} = 79.3651$$

The LINEST function tells us the uncertainty of the y-intercept is: $\pm$ 0.0027

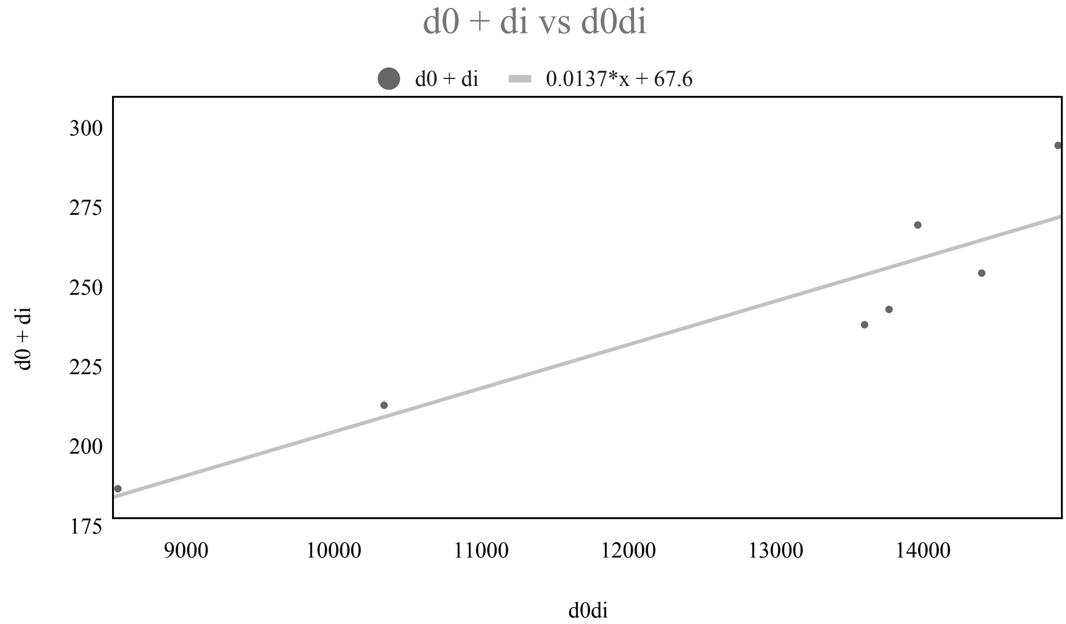

Figure 3.4: $d_0 + d_\mathrm{i}$ vs $d_0 d_\mathrm{i}$

In this case, the slope of our graph is: $\frac{1}{f}$
Focal Length:

$$0.0137^{-1} = 72.9927$$

The LINEST function tells us the uncertainty of the y-intercept is: $\pm$ 11.804

Comparing Focal Lengths

- Method 1: 79.3651 cm ± 0.0027
- Method 2: 72.9927 cm ± 11.804

Neither of our methods gave us a focal length similar to that of our rough estimate of 59cm, however the first method gave us a much more accurate answer with lower uncertainty, the second method has a closer value to our rough estimate but the uncertainty is relatively high.

Table 3: Spherometer Measurements

Parameter	Value (mm)	Uncertainty (mm)
a	2.49	± 0.5
h_1	1.25	± 0.5
h_2	1.28	± 0.5
$R_1 = \frac{a^2+{h_1}^2}{2h_1}$	3.11	± 1.43
$R_2 = \frac{a^2+{h_2}^2}{2h_2}$	3.06	± 1.38
f_1 (From Eq. 2)	2.97	± 0.5
f_1 (From Expt 1.)	20.12	± 0.0004

Using Eq 2:

$$\frac{1}{f} = (1.520 - 1)\left[\frac{1}{3.11} - \frac{1}{-3.06}\right]$$

$$\frac{1}{f} = 0.34$$

$$f = 2.97 \text{ cm}$$

Assuming n = 1.520, keeping in mind R_2 is negative since it is a convex lens.

Using Eq.2 to find n:

$$n = \frac{1}{f[\frac{1}{R_1} - \frac{1}{R_2}]} + 1$$

$$n = 1.0767$$

Uncertainty Calculations

$$a^2 + h^2 \rightarrow \Delta Y = \Delta A + \Delta B$$

$$(0.5)^2 + (0.5)^2 = \pm 0.5$$

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

$$R_1 : \frac{0.5}{7.7625} + \frac{2\times 0.5}{2\times 1.25} = 0.46 \times 3.11 = \pm 1.43$$

$$R_2 : \frac{0.5}{7.8385} + \frac{2\times 0.5}{2\times 1.28} = 0.45 \times 3.06 = \pm 1.38$$

$$\frac{1}{3.11+3.06} + \frac{0.010}{1.520} = 0.17 \times 2.97 = \pm 0.5$$

4 Conclusion:

We were able to determine the focal length of our convex lens and combined lenses to 20.1207 ± 0.0004 and 79.3651 ± 0.0027 respectively using Method 1, which seemed to produce the lowest uncertainty throughout the experiment.

We got an extremely different value for the focal length of our convex lens in experiment two, 2.97 cm $\pm$ 0.5. Which is likely due to how difficult it was to carry out the experiment and collect accurate results. In future it would be beneficial to collect a larger number of values for each variables and use an average, hopefully reducing errors made when carrying out the experiment.

Finally we determined the refractive index of the lens to be 1.0767 which would make it an incredibly transparent substance which refracts light very little, comparable to air. However seeing as our focal length in Exp. 2 varied widely from our rough estimate and our final value of Exp. 1, it is likely that this answer is also unreliable.

5 Appendix

Question: Whilst observing a real image on the screen for a convex lens and keeping everything else the same, what happens to the image distance if you:

• Increase the object distance d_0:	( ) increases, ✓ decrease, ( ) no effect.
• Decrease the focal length f of the lens:	( ) increases, ✓ decrease, ( ) no effect.
• Increasing the wavelength λ:	( ) increases, ( ) decrease, ✓ no effect.
• Decrease the size of the object:	( ) increases, ( ) decrease, ✓ no effect.