Experiment 4
The Half-Life of Barium-137

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B.Sc in Applied Physics.

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10:00 A.M - 1:00 P.M

Introduction:

This experiment involves what is known as "milking the cow", or a chemical reaction which occurs when eluting solvent is passed through a Cs/Ba. 137m isotope generator. Our goal is to record the count of the solution that has passed through the isotope generator using a Geiger counter measured against time time and graph it, which will hopefully give us some insight into the half life of Barium, this graph gives us multiple ways of figuring out the half life, we will carry out and explain the method behind both.

When the eluting solvent passes through the Cs-137, the Cs disintegrates and forms a new daughter nucleus which is barium in an excited state, represented by Ba-137m where the m denotes this excited state. This Ba-137m is not stable in this excited state and so it inevitably gives of energy to reach Ba-137. The reaction equations are as follows:

$$\text{Cs-137} \rightarrow \text{Ba-137m} +_{-1} e^0$$

$$\text{Ba-137m} \rightarrow \text{Ba-137} +_0 \gamma^0$$

The goal of this experiment is that the Geiger counter should be able to measure these extra particles of energy being given off, the ${}^{0}_{-1}e$ particle being an beta particle while the ${}^{0}_{0}\gamma$ particle denotes gamma radiation, as the time since beginning the measurement increases the geiger counter should detect an exponentially decaying amount of particles (or photons) being ejected.

Knowing the constant decay rate k should be able to help us in working out the half life of Barium-137 which is ultimately the goal of the experiment. We will use the relationship:

$$0.693 = kt^{1/2} \tag{1}$$

to help us find $t^{1/2}$ the half life of the sample.

Method:

The main apparatus for this experiment is the Isotope Generator kit which contains the Cs-137 sample, as well as eluting solution which is a solution made up of 0.9% NaCl in 0.04M of HCl with the rest being made up of deionized water. A Geiger-Muller (GM) counter which can record the count of radioactive decay. The rest of the equipment is made up of a retort stand, a Cassy-Lab 2 Sensor which sends the Geiger-Muller counter data to a laptop which displays the count in real time, as well as a test tube to hold the isotope generator over and to drop the eluate into so that it can be picked up by the Geiger-Muller counter and two beakers, one to hold the unused eluting solution and one for waste eluate which has to sit for 30 minutes before it can be properly disposed of (this ensures that the barium has decayed enough that it is no longer a potential danger).

We start off by assembling the retort stand in such a way where the test tube is clamped slightly below eye level, this way we can have the isotope generator which is placed above the test tube directly at eye level and we are being as accurate as possible while carrying out the experiment. Meanwhile the Geiger-Mullter counter is placed directly below the test tube facing up towards it, its important that we don't place the test tube directly on the counter as this may distort the readings however due to the inverse square law that describes the relationship between intensity and distance for radioactivity, we want to get the counter as close as possible for the best reading, a distance of 0.5 cm between them should be sufficient.

We connect the Cassy-Lab 2 Sensor to both the Geiger counter as well as the laptop with the software preinstalled before setting the parameters of our graph within the software. We want to to generate a graph which represents the count value with respect to time, for this to happen we ned to select a measuring range which in our case ended up being 100 s^{-1}, a Gate time which we selected to be 12 seconds and an interval of 100 miliseconds. It's important that we have our count value on the y-axis and our time value on the x-axis so we end up with a graph that is representative of the count value over time, and also makes our analysis easier later on in the experiment.

As mentioned in the introduction, the method by which is the excited barium 137 atoms are created is by pouring 2-3ml of eluting solution through our isotope generator which has the chemical reaction listed above with the Cs-137 sample inside, one the eluate has left the generator and dropped down into the bottom of the test tube the measurement is immediately started and left to run forming a graph for roughly 1000 seconds.

Once the measurement is complete the data and graphs are saved and the eluate in the test tube is poured into the waste beaker and left for 30 minutes, at this point the experiment is ready to be repeated.

Results:

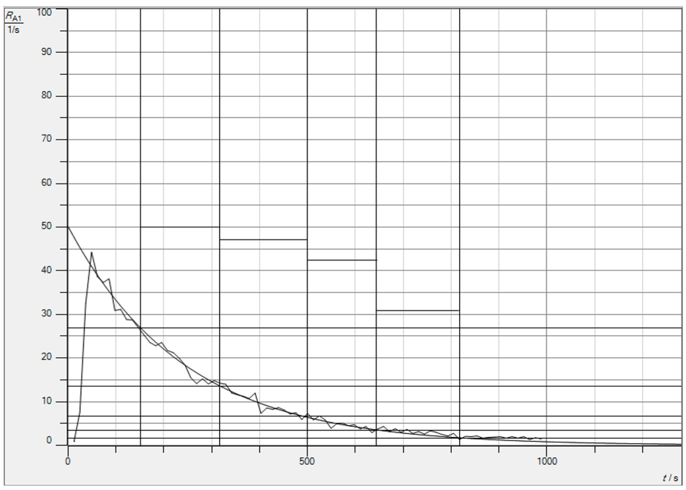
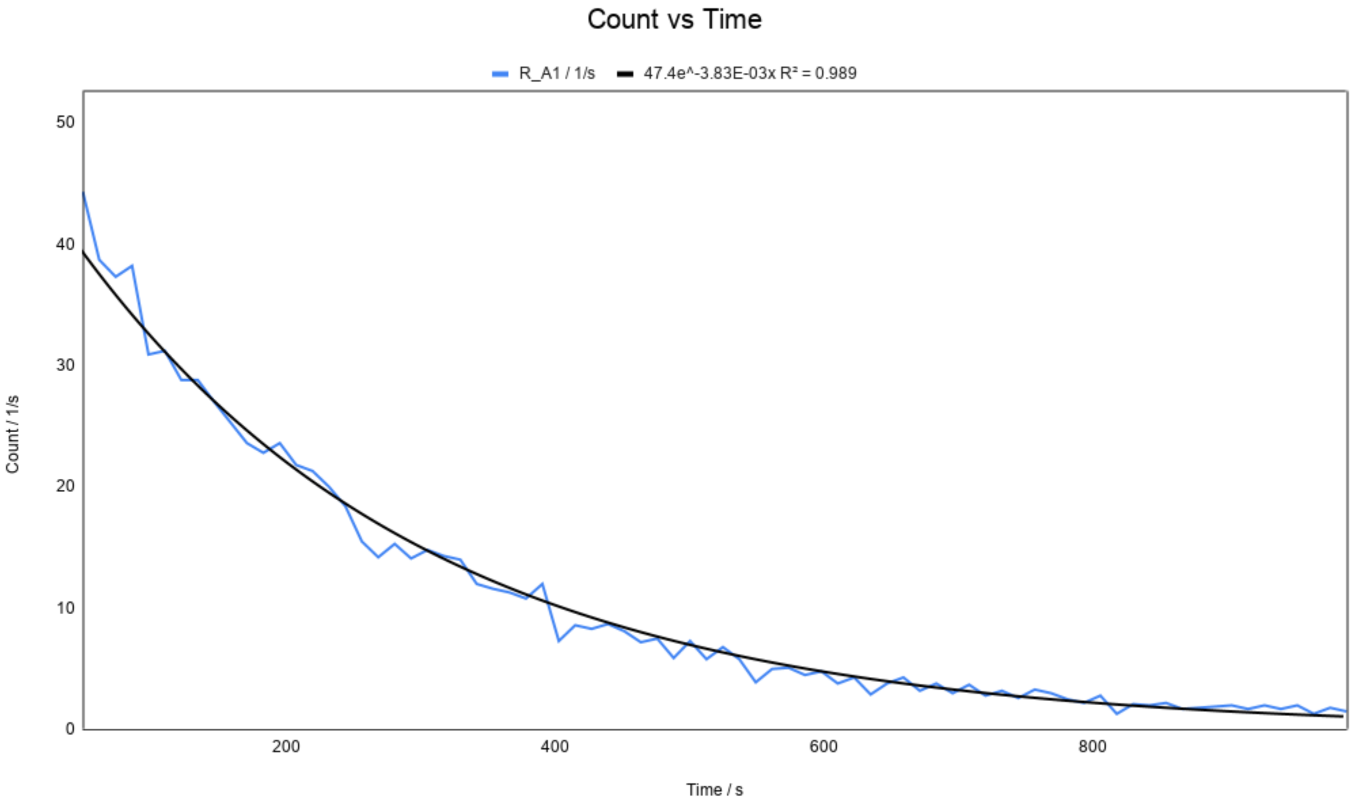
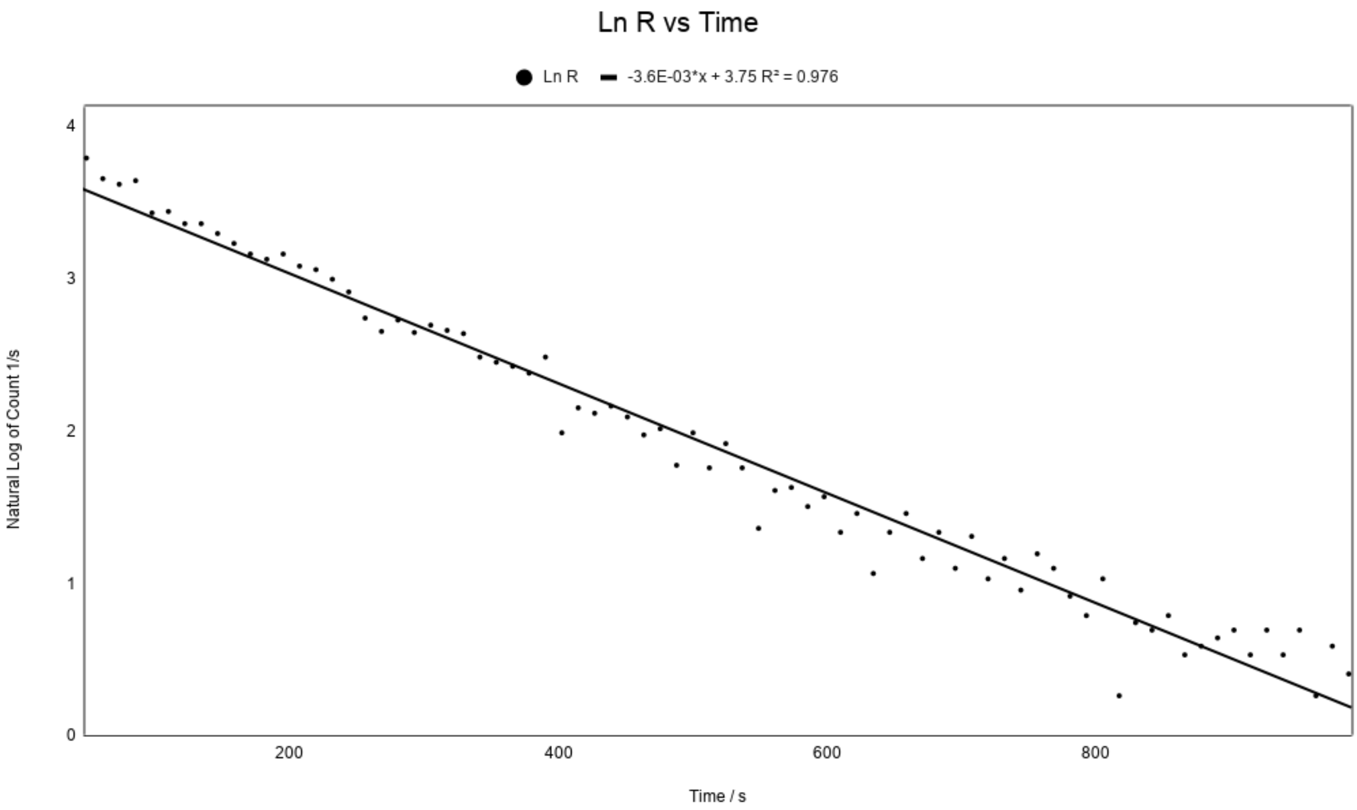


Figure 1: Graph of attempt 1 with half life analysis



Graph 1: Count vs Time (attempt 1)



Graph 2: Ln Count vs Time (attempt 1)

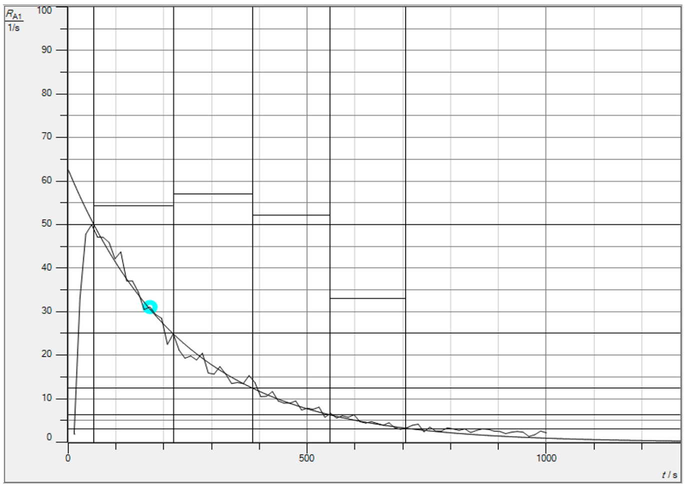
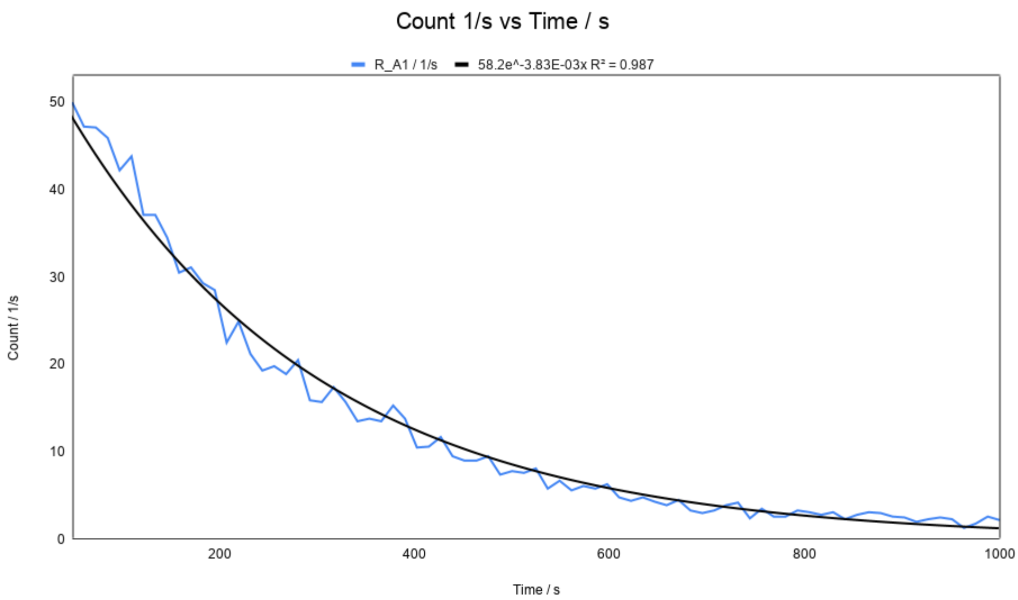
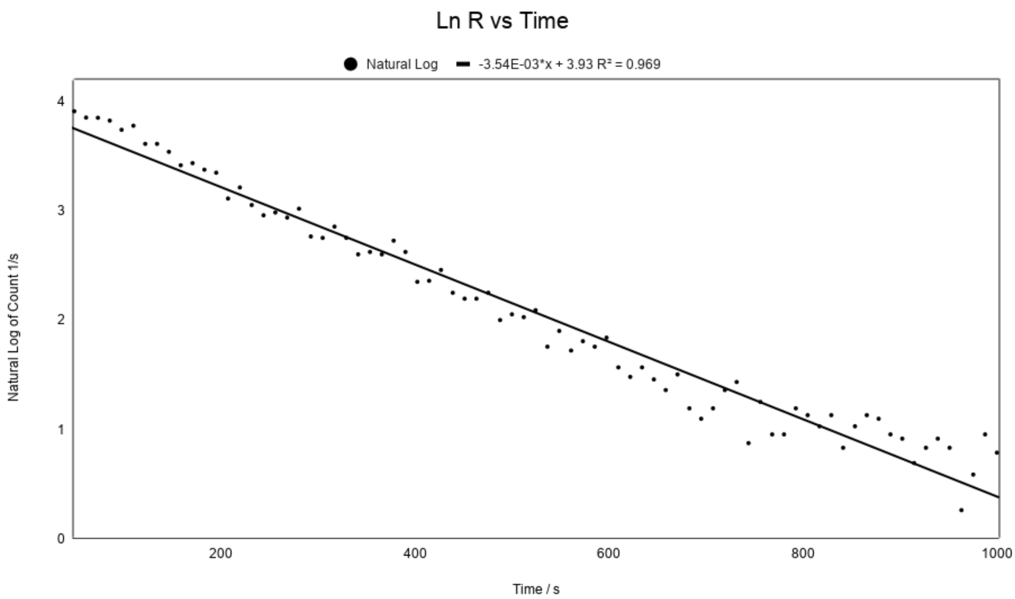


Figure 2: Graph of attempt 2 with half life analysis



Graph 3: Count vs Time (attempt 2)



Graph 4: Ln Count vs Time (attempt 2)

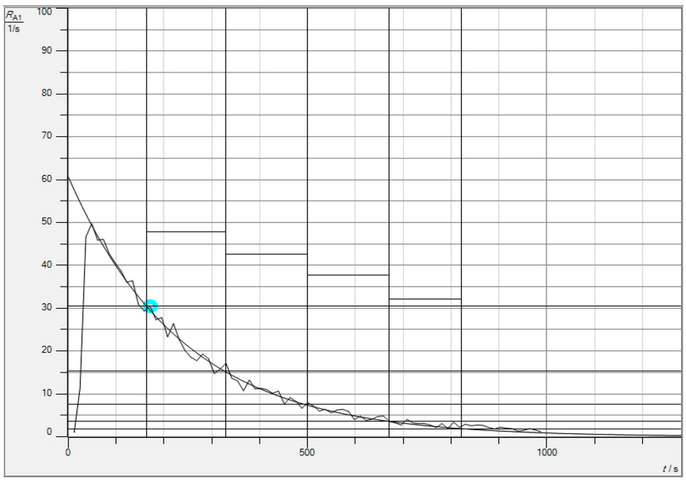
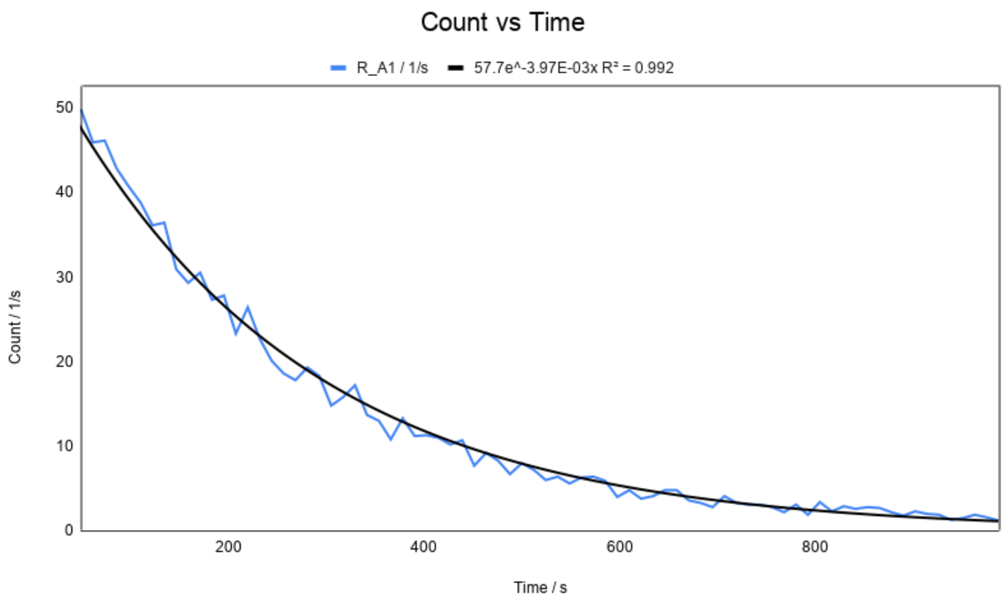
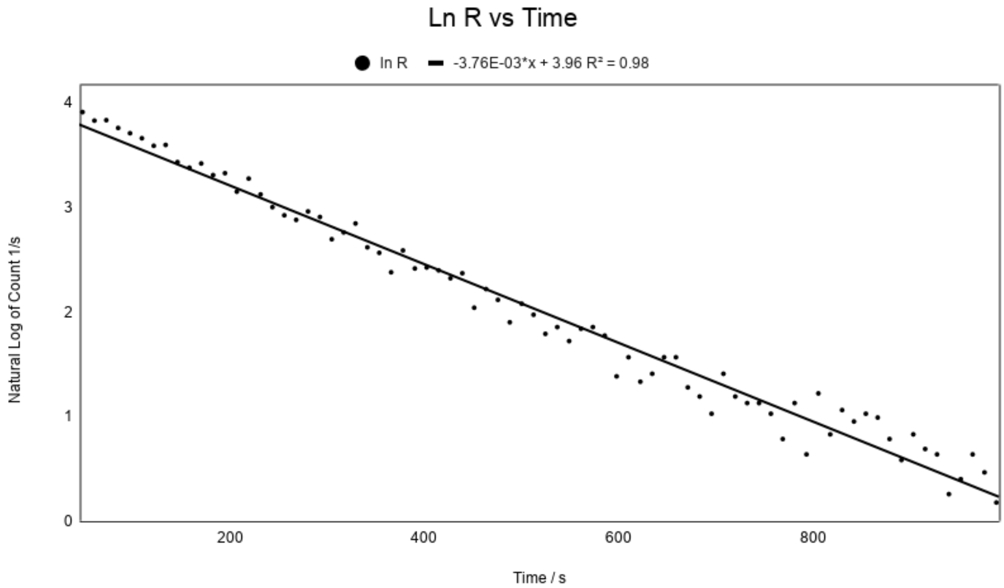


Figure 2: Graph of attempt 3 with half life analysis



Graph 3: Count vs Time (attempt 3)



Graph 4: Ln Count vs Time (attempt 3)

Analysis:

Our first method of determining the half life for Barium 137 is by taking 5 well defined points along the graph and measuring the 4 distances between the 5 points. These distances are represented by horizontal lines placed above the exponential curve in Fig. 1, 2 and 3.

Figure 1.

From R = 27 to R = 13.5	$\Delta t = 165.3$ seconds
From R = 13.5 to R = 6.75	$\Delta t = 183.1$ seconds
From R = 6.75 to R = 3.375	$\Delta t = 145.8$ seconds
From R = 3.375 to R = 1.6875	$\Delta t = 172.4$ seconds
Average value of Δ t	$\Delta t = 166.65$ seconds

Figure 2.

From R = 50 to R = 25	$\Delta t = 165.3$ seconds
From R = 25 to R = 12.5	$\Delta t = 165.3$ seconds
From R = 12.5 to R = 6.25	$\Delta t = 161.8$ seconds
From R = 6.25 to R = 3.125	$\Delta t = 158.2$ seconds
Average value of Δ t	$\Delta t = 162.65$ seconds

Figure 3.

From R = 30.5 to R = 15.25	$\Delta t = 165.3$ seconds
From R = 15.25 to R = 7.625	$\Delta t = 170.7$ seconds
From R = 7.625 to R = 3.8125	$\Delta t = 168.9$ seconds
From R = 3.8125 to R = 1.90625	$\Delta t = 151.1$ seconds
Average value of Δ t	$\Delta t = 164$ seconds

Another way we can determine the half life for Barium is by taking the slope of our Ln R vs Time graphs as our value k and by using Eq. 1 we can determine the half life $t^{1/2}$. Manipulating Eq.1 we get:

$$\frac{0.693}{k} = t^{1/2}$$

The half lives are as follows:

Attempt 1

$$t^{1/2} = \frac{0.693}{3.6 \times 10^{-3}} = 192.5 \text{ seconds}$$

Attempt 2

$$t^{1/2} = \frac{0.693}{3.54 \times 10^{-3}} = 195.76 \text{ seconds}$$

Attempt 3

$$t^{1/2} = \frac{0.693}{3.76 \times 10^{-3}} = 184.31 \text{ seconds}$$

We can also calculate a half life for Barium based on the Count vs Time graphs, the equation of the exponential line follows the relationship:

$$N/N_o = e^{-kt} \tag{2}$$

So the power of the exponential is our k value in this circumstance. The half lives for each exponential graph are as follows:

Attempt 1

$$t^{1/2} = \frac{0.693}{3.83 \times 10^{-3}} = 180.94 \text{ seconds}$$

Attempt 2

$$t^{1/2} = \frac{0.693}{3.83 \times 10^{-3}} = 180.94 \text{ seconds}$$

Attempt 3

$$t^{1/2} = \frac{0.693}{3.97 \times 10^{-3}} = 174.56 \text{ seconds}$$

In an effort to improve overall accuracy we can try obtaining the average value and standard deviation of these found values:

Average value:

$$\frac{180.94 + 180.94 + 174.56}{3} = 178.81 \text{ seconds}$$

Using the equation:

$$\sigma = \sqrt{\frac{\sum(x-\mu)^2}{N}} \tag{3}$$

Where x is the values found above, μ is the average value calculated above and N is the number of values we have, 3. We can find the standard deviation.

Standard Deviation:

$$\sigma = \sqrt{\frac{(180.94 - 178.81)^2 + (180.94 - 178.81)^2 + (174.56 - 178.81)^2}{3}}$$

$$\sigma = 3.01\ s$$

This implies that 68% of the time our value for $t^{1/2}$ will fall in the range of 178.81 $\pm$ 3.01, and 95% of the time our value for $t^{1/2}$ will fall between the range of 178.81 $\pm$ 6.02.

Even with this 95% thresh hold, the upper end value of 184.83 is still lower than two of the half lives calculated using the Ln R vs Time graph, with only one of the values falling within the range. Overall the Ln R graph method seems to produce substantially higher half life values than the other methods attempted in this report.

Manipulating Eq. 1 to isolate k we get:

$$\frac{0.693}{t^{1/2}} = k$$

Subbing in 178.81 for $t^{1/2}$ we get:

$$k = \frac{0.693}{178.81} = 3.88 \times 10^{-3} s^{-1}$$

to find the uncertainty associated with this value we can use:

$$\frac{\Delta k}{k} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta t^{1/2}}{t^{1/2}}\right)^2} \tag{4}$$

$$\Delta k = \sqrt{\left(\frac{0}{0.693}\right)^2 + \left(\frac{3.01}{178.81}\right)^2} \times 3.88 \times 10^{-3} = 0.07$$

So our determined value for k is:

$$k = 3.88 \times 10^{-3} \pm 0.07 \times 10^{-3} s^{-1}$$

We can now compare all of the half life values found to the actual known half life for a sample of Barium-137. The given value for the half life of Barium-137 is 2.551 minutes or 153.06 seconds.

We can find the percentage discrepancy using the following equation:

$$\frac{|\text{Observed} - \text{Actual}|}{\text{Actual}} \times 100 \tag{5}$$

The discrepancies are as follows:

<u>Graph analysis Figure 1</u>

$$\frac{|166.65 - 153.06|}{153.06} \times 100 = 8.88\%$$

<u>Graph analysis Figure 2</u>

$$\frac{|162.65 - 153.06|}{153.06} \times 100 = 6.27\%$$

<u>Graph analysis Figure 2</u>

$$\frac{|164 - 153.06|}{153.06} \times 100 = 7.15\%$$

We can now find the discrepancies for the Ln R vs Time graph estimates.

<u>Ln R vs Time graph attempt 1</u>

$$\frac{|192.5 - 153.06|}{153.06} \times 100 = 25.77\%$$

<u>Ln R vs Time graph attempt 2</u>

$$\frac{|195.76 - 153.06|}{153.06} \times 100 = 27.90\%$$

<u>Ln R vs Time graph attempt 3</u>

$$\frac{|184.31 - 153.06|}{153.06} \times 100 = 20.42\%$$

Finally we can find the discrepancy for each of the exponential graph half lives, as well as see how accurate our average value turned out to be:

<u>Exponential graph analysis attempt 1</u>

$$\frac{|180.94 - 153.06|}{153.06} \times 100 = 18.22\%$$

<u>Exponential graph analysis attempt 2</u>

$$\frac{|180.94 - 153.06|}{153.06} \times 100 = 18.22\%$$

<u>Exponential graph analysis attempt 3</u>

$$\frac{|174.56 - 153.06|}{153.06} \times 100 = 14.05\%$$

Average exponential analysis half life:

$$\frac{|178.81 - 153.06|}{153.06} \times 100 = 16.82\%$$

We can clearly see that the discrepancy between the average half life and the actual half life is far greater than the uncertainty calculated.

Discussion:

Its important to point out that there is a clear difference between the graph that was created in real time during the experiment and the graph that was later plotted for the exponential equation, this is in reference to the original graph beginning at zero before spiking to a high value that isn't existant in the reproduced graph. This is due to some advice I received from a lab technician during the experiment, it was recommended that, since I was carrying out the experiment alone and it was impossible to pour the eluting solvent and press the start button at the same time, it would be best to start the measurement a few seconds early and just discard any readings obtained before the eluting solution had been poured. This method proved to be extremely effective and time efficient, the original graph is only included for the sake of total transparency.

Although every effort was made to ensure that background radiation was kept to a minimum, its clear from the graph that spikes are present in the count values which implies interference of some kind was taking place throughout the entire experiment. A suggestion for reducing this in future maybe to ensure that the test tube and counter are housed in something which would ensure that the only source of radiation is coming from the test tube directly above.

A large part of this experiment is ensuring that the counter and test tube are as close as reasonably possible without touching for reasons mentioned in the method section, however throughout the course of the experiment over multiple measurements the test tube has to be removed from the clamp so that the waste can be placed into the waste beaker and then readjusted into place. It's important to measure the distance between the bottom of the test tube and the top of the counter after this readjustment however small differences in distance and angle may occur. In an effort to increase accuracy it might be wise to make the counter holder and the test tube holder fixed in some way so that no matter how many times the experiment is ran the apparatus is always identical, and the results will at least be more consistent- if not more accurate.

Conclusion:

In this report we attempted to determine the half life of Barium-137 from a single experiment using multiple different methods and verifying the discrepancy between them and the actual value for the half life. It was determined that the first method which involved measuring the distance between 5 distinct points on the graph had the highest accuracy and precision out of all the methods, we obtained 3 values for the half life which had a discrepancy of 8.88%, 6.27% and 7.15% respectively. We determined that the method which involved graphing the natural log of the radioactive count vs time had the greatest discrepancy from the actual half life, with discrepancies as high as 27.90%. We were also able to determine a value for k using the exponential graph method and also calculate the uncertainty associated with it, which worked out to be $k = 3.88 \times 10^{-3} \pm 0.07 \times 10^{-9}$.