Determination of the AC Mains Frequency Using a Sonometer

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1 Introduction:

The main operational theory of a sonometer revolves around the idea that a string pulled taut that is given a certain force will vibrate. Whether you pluck the string (this is the operation carried out with a guitar), provide frictional force by rubbing the string against another surface (this is the operation behind a violin) or as is the case with this experiment, provide a force in the form of electrical current.

Resonant Frequency just refers to the correct frequency required to make the string vibrate with the greatest amplitude, every object has a natural frequency associated with it and this frequency can be determined by first applying enough force to make the object vibrate and secondly getting the object to a point where it has the greatest amplitude.

In this experiment we will carry out finding the resonant frequency of a string with current flowing through it at different tensions by varying the length of a bridge placed below, once we find the point where the string vibrates at the highest possible amplitude we can determine that the point the bridge is located at is the strings resonant frequency. using the equation:

$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \tag{1}$$

Where T = tension, μ = mass / unit length and L = length

2 Experimental Setup:

The experiment consisted of a sonometer, which is a long plank of wood with a string pulled taut across it with the ability to vary the strings tension by increasing or decreasing the weight acting upon the string at one end. A power supply which acts as our main source of AC current in the experiment. A rheostat which allows us to control the amount of current flowing through the wire at any one time and a magnet, which gives the wire a small force to enable it to start resonating due to the cross sectional force caused by the intersection of a magnetic field and an electric field flowing in different directions. The sonometer also has three bridges stretched across it, two stationary bridges and one bridge which can be moved to find the exact length at which resonance occurs.

Due to the string being pulled taut and parts of the experiment requiring increasing the tension of the string it was mandatory to wear eye protection in the interest of protecting our eyes from any potential breaking of the string causing the high tension string to jump erratically and cause harm. This is also the reason behind caution being taken when increasing the weight acting on the string.

3 Results & Data Analysis:

Table 1: First set of sonometer results

Mass (kg)	Resonant Length (L / m)	L^2 (m^2)
0.5	0.612	0.375
0.6	0.672	0.452
0.7	0.750	0.563
0.8	0.789	0.623
0.9	0.832	0.692
1	0.881	0.776
1.1	0.926	0.857
1.2	0.950	0.903

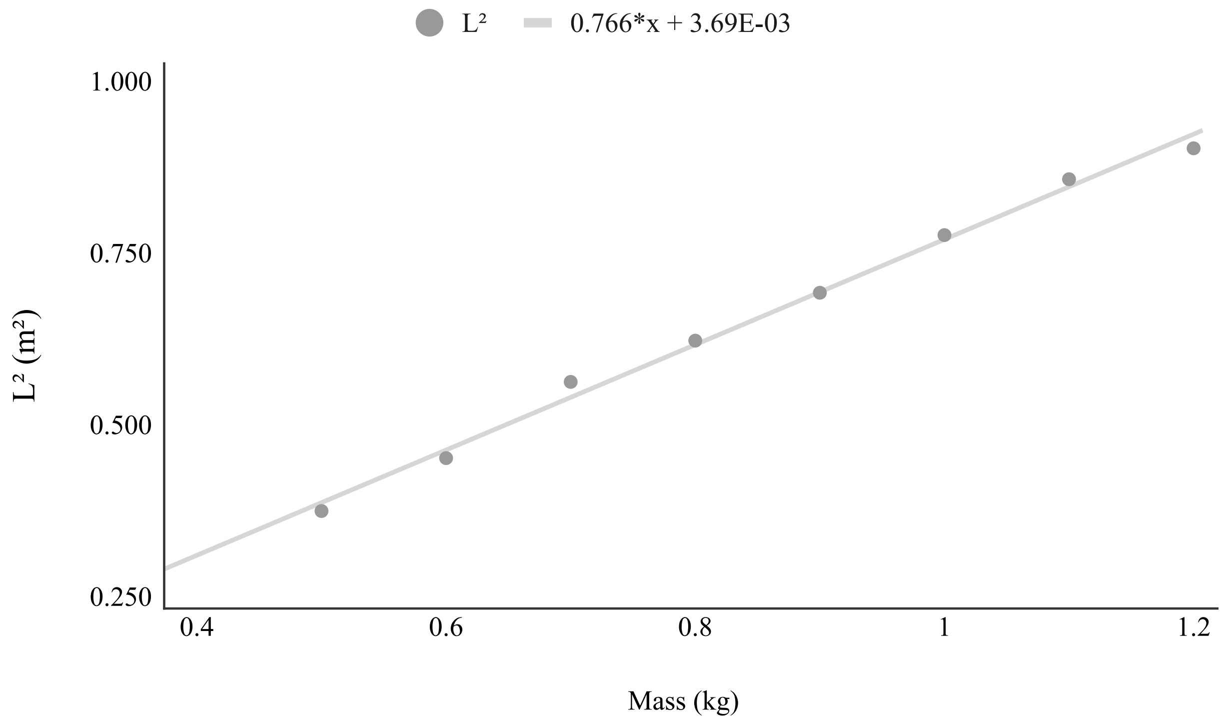

Figure 3.1: Mass (kg) vs L^2

We can determine the slope of our line from the graph above. Our slope works out to be 0.766.

$$\boxed{0.766\ \text{kg/m}^2}$$

Table 2: Second set of sonometer results

Mass (kg)	Resonant Length (L / m)	L^2 (m^2)
0.5	0.636	0.404
0.6	0.682	0.465
0.7	0.739	0.546
0.8	0.796	0.634
0.9	0.834	0.696
1	0.881	0.776
1.1	0.917	0.841
1.2	0.953	0.908

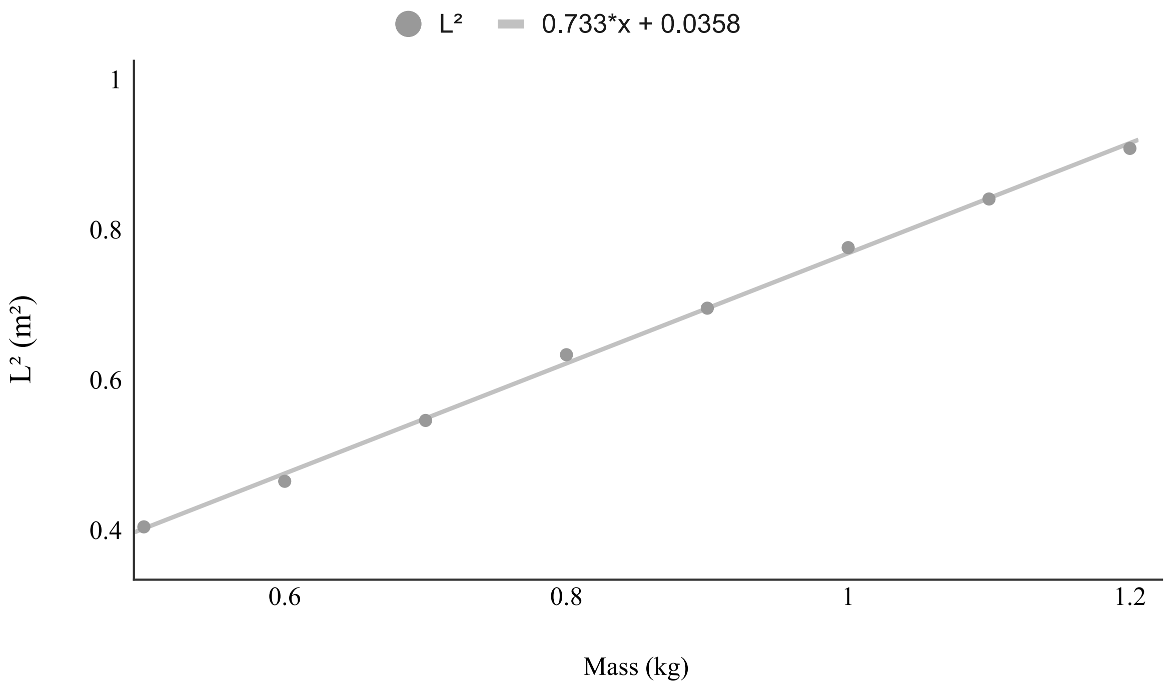

Figure 3.2: Mass (kg) vs L^2

We can determine the slope of our line from the graph above. Our slope works out to be 0.733.

$$\boxed{0.733 \text{ kg/m}^2}$$

Table 3: Diameter of the wire

	Diameter (m)
1	4.9×10^{-5}
2	4.4×10^{-5}
3	4.5×10^{-5}
4	4.5×10^{-5}
5	4.4×10^{-5}
6	4.3×10^{-5}
7	4.4×10^{-5}
8	4.2×10^{-5}
9	4.4×10^{-5}
10	4.4×10^{-5}
Avg:	4.4×10^{-5}

We found the average of our values using the equation:

$$\bar{x} = \frac{\sum x}{n} \tag{2}$$

Where $\bar{x}$ = average, x = our diameter values and n = total number of values.
Now that we have a value for $\bar{x}$, we can use this value to help find the standard deviation (σ) using the equation:

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} \tag{3}$$

We get $\sigma = 1.74x10^{-6}$

Assuming that our standard deviations can be used as values for our uncertainty our values change accordingly:

Table 4: Diameter of the wire with uncertainty

	Diameter (m)
1	$4.9 \times 10^{-5} \pm 1.74x10^{-6}$
2	$4.4 \times 10^{-5} \pm 1.74x10^{-6}$
3	$4.5 \times 10^{-5} \pm 1.74x10^{-6}$
4	$4.5 \times 10^{-5} \pm 1.74x10^{-6}$
5	$4.4 \times 10^{-5} \pm 1.74x10^{-6}$
6	$4.3 \times 10^{-5} \pm 1.74x10^{-6}$
7	$4.4 \times 10^{-5} \pm 1.74x10^{-6}$
8	$4.2 \times 10^{-5} \pm 1.74x10^{-6}$
9	$4.4 \times 10^{-5} \pm 1.74x10^{-6}$
10	$4.4 \times 10^{-5} \pm 1.74x10^{-6}$

We measured the mass of the 20m string to be 2.493×10^{-2} kg ± 0.0005, therefore we can calculate the mass per unit length of the string using:

$$\mu = \frac{Mass}{Length} \tag{4}$$

where μ = Mass per unit length
Using Eq. 4 we get:

$$1.25 \times 10^{-3} = \frac{2.493 \times 10^{-2}}{20}$$

The mass per unit length works out to be 1.25×10^{-3} kg/m, we know the balance's uncertainty is ± 0.0005
The value of the length was given to us and we can assume it has no uncertainty associated with it. Therefore we can determine the uncertainty of the mass per unit length is simply. ± 0.0005

Therefore our value for μ is:

$$\boxed{1.25 \times 10^{-3} kg/m}$$

Now we can finally calculate a value for f_0 using

$$L^2 = \frac{g}{4\mu {f_0}^2} m \tag{5}$$

Using our first set of data from our very first table we know that when $m = 0.5\ kg$, $L^2 = 0.375$
Manipulating Eq.5 we get:

$$f_0 = \sqrt{\frac{g\,m}{L^2(4\mu)}}$$

This gives us the following:

$$51.15 = \sqrt{\frac{9.81 \times 0.5}{0.375 \times (4(1.25 \times 10^{-3}))}}$$

This gives us a value of 51.15 Hz,
We can repeat this process for all the values in Table 1 and Table 2 and work out an average:

Table 5: First f_0 values

Mass (kg)	L^2 (m^2)	f_0
0.5	0.375	51.15
0.6	0.452	51.03
0.7	0.563	49.39
0.8	0.623	50.19
0.9	0.692	50.51
1	0.776	50.28
1.1	0.857	50.18
1.2	0.903	51.06
	Average:	50.47

Table 6: Second f_0 values

Mass (kg)	L^2 (m^2)	f_0
0.5	0.404	49.28
0.6	0.465	50.32
0.7	0.546	50.15
0.8	0.634	49.76
0.9	0.696	50.37
1	0.776	50.28
1.1	0.841	50.66
1.2	0.908	50.92
	Average:	50.22

From our two sets of results we can determine that the frequency of the AC is about 50Hz Only one of the values used in our calculations has an uncertainty associated with it, that being the uncertainty of the mass per unit length of ± 0.0005 Therefore our final value is

$$50 \text{ Hz} \pm 0.0005$$

This value is entirely consistent with the frequency associated with AC current in Ireland as of November, 2019.[1]

We can also work out the mass per unit length of an object using a different method if we don't know the mass and length of the string but we do know the diameter. In this case we have a steel wire with the double the diameter of the one we measured, therefore it has a diameter of: 8.8×10^{-5}m
Using the equation:

$$cross\text{–}sectional\ area = \pi(d/2)^2 \tag{6}$$

We can find the cross sectional area of this wire like so:

$$6.08 \times 10^{-9} = \pi(\tfrac{8.8\times10^{-5}}{2})^2$$

Since we know that this wire is also made out of steal we can determine that the density is the given value $7.9 \times 10^3 kgm^{-3}$
Therefore we can use the equation:

$$\mu = density \times cross\text{–}sectional\ area \tag{7}$$

To work out our mass per unit length of this new wire. The calculation is as follows:

$$4.8 \times 10^{-5} = (7.9 \times 10^3) \times (6.08 \times 10^{-9})$$

We can now use Eq. 5 to find L^2 and just square root our answer:

$$10.2 = \tfrac{9.81}{(4(4.8\times10^{-5}))(50)^2}(0.5)$$

$L^2 = 10.2$ so therefore the resonant length of a steel wire with double the diameter is 3.2 m

Using the LINEST function on our 2 graphs we can work out that the experimental error's of the two graphs are 0.02 and 0.01 respectively.

4 Conclusion:

We managed to find the AC mains frequency, which turned out to be 50 Hz $\pm$ 0.0005
We also discovered the mass per unit length of a wire using two different methods.

5 Appendix:

Answer the question:
(i) Why do suspension bridges sometimes vibrate with dangerously large amplitudes?

"All objects have a natural frequency or set of frequencies at which they vibrate"[2] however most objects are stable and remain unperturbed during every day use. Suspension bridges are no exception to this and also have a natural frequency at which they would resonate. Although uncommon, it is possible for suspension bridges to feel the affects of resonance and vibrate with dangerously large amplitudes at times when the wind acts on the bridge at just the right frequency.

(ii) What is the purpose of the rheostat in the circuit?

As previously mentioned the rheostat acts as a variable resistor which we can use to control the amount of current running through the wire at any point in time. Although we had the ability to vary the rheostat it was mostly kept at one particular resistance just to ensure that there was in fact a current and that the current wasn't so great it was causing the wire to heat up.

(iii) Predict that effect on the resonance when:
(a) the tension in the wire is increased

We know that as tension is increased so too does frequency, thus we can conclude that if the tension in the wire is increased the resonant frequency of the wire will also increase.

(b) the density of the wire is increased

Increasing the density of a string has a negative affect on the frequency of the string, so we know that increasing the density of the wire will reduce the resonant frequency of the wire.

(c) the driving frequency is increased.

As the driving frequency increases we would expect to see the amplitude of the waves get smaller as the energy is transferred less efficiently.

(iv) The steel B string of an acoustic guitar is 63.5cm long and has a diameter of 0.406mm (16 gauge). Under what tension must the string be placed to give a frequency for transverse waves of 247.5 Hz? The density of steel is 7800 kg/m3. Assume that the string vibrates in the fundamental mode.

With the information given we can work out the mass per unit length using Eq. 5 and 6 Using Eq. 6 we get:

$$\pi(\frac{(0.000406)}{2})^2 = 1.29 \times 10^{-7}$$

Now we multiply our value for the cross sectional area by the density value given using Eq. 6:

$$(1.29 \times 10^{-7}) \times (7800) = 1 \times 10^{-3}$$

The reason for working out the mass per unit length is that we need it as a variable in Eq.5 that we will be manipulating to help find the tension needed. Manipulating the equation gives us

$$(\frac{f_0}{\frac{1}{2L}})^2 \mu = T$$

Using this equation we get:

$$(\frac{247.5}{\frac{1}{2(0.635)}})^2 (1 \times 10^{-3}) = 98.80$$

Therefore the required tension is 98.80 Nm^{-1} (Newtons per meter)

References

[1] Electric Ireland - what is the standard voltage in Ireland?

[2] T. H. the Physics Classroom. Sound waves and music - lesson 4 - resonance and standing waves: Natural frequency.