Determination of Refractive Index Using Snell's Law

Jamie Lee Somers,
(DC171) B.Sc in Applied Physics.

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1 Introduction:

The main aims of this experiment were to be able to figure out the refractive index of a substance using a well understood relationship between the angle of incidence θ_i and the angle of refraction θ_r, This relationship is most notably known as snell's law and it is described as follows:

$$n_1\ Sin\theta_1 = n_2\ Sin\theta_2 \tag{1}$$

Where Sinθ_1 is the angle of incidence and n$_1$ is the refractive index of the initial medium. Sinθ_2 is the angle of refraction and n$_2$ is the refractive index of the secondary medium.

The importance of refractive index is its ability to describe how fast light is travelling in a given medium compared to how fast light travels in the absence of any medium (in a vacuum), therefore the refractive index is often used to describe how much slower light travels through different mediums. As such, refractive index is mathematically described as the ratio of lights speed in a vacuum to lights speed in a given material:

$$n = \frac{c}{v} \tag{2}$$

Where c is the speed of light in a vacuum and v is the speed of the light within a medium.

The reason that this change in speed results in a change in angle is due to the fact that light can be considered an electromagnetic wave, the changing in the speed of the wave between the two mediums results in an apparent bend in the wave as it passes from on side to another, this bend can be measured by comparing the angles at which the ray of light entered the medium and the angle at which the ray of light exited the medium.

However, at certain angles this bending has much greater consequences for the ray of light, one of the most well known angles when it comes to refraction is 90°, this angle is named the critical angle. This is due to the fact that if a ray of light goes from a medium with a higher refractive index to a lower refractive index $n_1 > n_2$ All of the rays are refracted away from the normal, Certain angles of incidence's will cause the ray to create a right angle when trying to exit the medium. The final result of this phenomenon is that no light will ever leave the medium past the critical angle.

Finding the angle of incidence of a medium that results in the critical angle being found is another method of determining the refractive index of a material. using the equation:

$$sin\theta_\mathrm{c} = \frac{1}{n} \tag{3}$$

Where sinθ_c is the angle of incidence when no light leaves the material.

Objects commonly associated with the bending of light are lenses, which can be both concave and convex in nature and can also exist as double sided in the form of converging and diverging.

All lenses have common equations which can be used such as:
The lens formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \tag{4}$$

Where u is the distance from the object to the lens, v is the distance from the lens to the image and f is the focal length of the lens.
Magnification formula

$$\frac{v}{u} = m \tag{5}$$

where m is the magnification of the image by the mirror.

Despite having some formulae in common, each of the lenses have their own unique characteristics on how the interact with objects and form images.

Diverging Lens

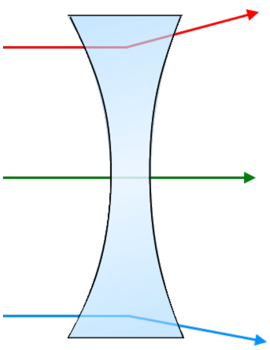

Figure 1.1: Diverging Lens

Diverging lenses will always produce a virtual upright image, these lenses cannot produce a real image and the image is always diminished.

The reason Diverging lenses cannot produce real images is due to the way the rays of light are bent away from each other and are never able to intersect to form a real image. Which is visible in Fig. 1.1
Diverging lenses are often used on flashlights to magnify the lights area.

Converging Lens

Converging lenses can produce both a real and virtual, magnified, upright image, however an object outside the lenses focal length will create a real image through the real intersection of rays

This can be seen in Fig. 1.2 where the rays of light intersect.
Converging lenses are often used in magnifying glasses due to their ability to magnify images.

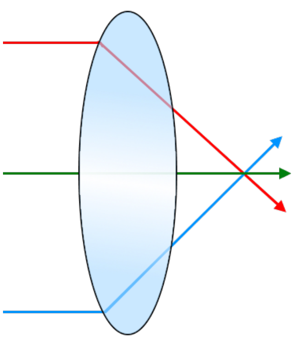

Figure 1.2: Converging Lens

2 Method:

Experiment 1: Measuring the refractive index of Perspex using Snell's Law

1. The perspex block was placed on a rotating table, so that the flat straight surface of the block was perfectly aligned with the two 90° markers located on the rotating table.

2. The laser was switched on so that it passed through both the 0° and the perspex block in a perfectly straight line. Entering the flat surface of the block and exiting the rounded surface.

3. Once all of these preliminary conditions were met the table was rotated 5° between the range of 15° and 65° making sure to measure the angle θ_{r} each time.

4. After 11 θ_{i} and 11 θ_{r} values were found, the experiment was ended and the results were tabulated and plotted on a graph. The slope of the graph can be inverted to find the refractive index of the medium.

Experiment 2: Measurement of the refractive index of a block of Perspex using a light-ray traveling from Perspex into air, and determination of the critical angle.

1. The perspex block was placed on a rotating table, so that the flat straight surface of the block was perfectly aligned with the two 90° markers located on the rotating table.

2. The laser was switched on so that it passed through the 0° and the perspex block in a perfectly straight line. Entering the rounded surface of the block and exiting the flat surface.

3. Once all of these preliminary conditions were met the table was rotated 5° between the range of 15° and 45° making sure to measure the angle θ_{r} each time.

4. These 7 results were used to determine the refractive index

5. By now the light beam was no longer leaving the Perspex block and we had to work backwards to determine when the internal refraction occured.

6. An Angle was found the the block was rotated in the opposite direction to verify this result.

3 Results and Analysis:

This experimental data was calculated using a manipulated form of Eq. 1 to determine the refractive index of n_2:

$$\frac{(1)Sin\theta_{\text{i}}}{Sin\theta_{\text{r}}} = n_2$$

Where $n_1 = 1$, the refractive index of air.

Table 1: Experiment 1: Snell's Law Results

Angle of incidence Sin θ_i	Angle of refraction Sin θ_r	Refractive index (n)
Sin(15°) = 0.258819	Sin(10°) = 0.173648	1.49
Sin(20°) = 0.342020	Sin(13°) = 0.224951	1.52
Sin(25°) = 0.422618	Sin(17°) = 0.292372	1.45
Sin(30°) = 0.5	Sin(20°) = 0.342020	1.46
Sin(35°) = 0.573576	Sin(23°) = 0.390731	1.47
Sin(40°) = 0.642788	Sin(26°) = 0.438371	1.47
Sin(45°) = 0.707107	Sin(27°) = 0.453990	1.56
Sin(50°) = 0.766044	Sin(31°) = 0.515038	1.49
Sin(55°) = 0.819152	Sin(34°) = 0.559193	1.46
Sin(60°) = 0.866025	Sin(36°) = 0.587785	1.47
Sin(65°) = 0.906308	Sin(39°) = 0.629320	1.44
	Average Refractive index	1.48

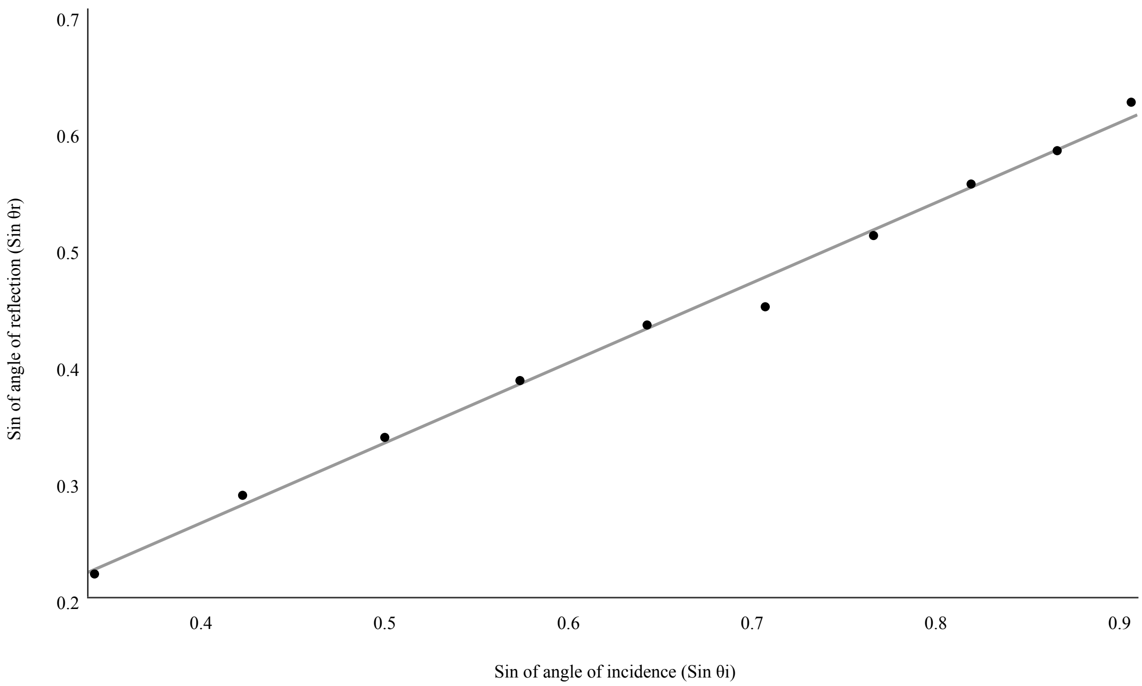

Figure 3.1: Angle of incidence Sin θ_i vs Angle of refraction Sin θ_r

This experimental data is slightly different as instead of Eq. 1 giving us the refractive index, instead we get:

$$\frac{Sin\theta_{\text{i}}}{Sin\theta_{\text{r}}} = \frac{n_{\text{air}}}{n_{\text{perspex}}} = \frac{1.00}{n_{\text{perspex}}} \tag{6}$$

Therefore the values we get from our calculations will have to be inverted in order to find the refractive index.

Table 2: Experiment 2: Snell's Law Results

Angle of incidence Sin θ_{i}	Angle of refraction Sin θ_{r}	Inverse Refractive index ($\frac{1}{n}$)	Refractive index (n)
Sin(15°) = 0.258819	Sin(23°) = 0.173648	0.6623967896	1.51
Sin(20°) = 0.342020	Sin(30°) = 0.224951	0.6840402867	1.46
Sin(25°) = 0.422618	Sin(37°) = 0.292372	0.7022394681	1.42
Sin(30°) = 0.5	Sin(48°) = 0.342020	0.6728163648	1.49
Sin(35°) = 0.573576	Sin(58°) = 0.390731	0.6763489464	1.48
Sin(40°) = 0.642788	Sin(72°) = 0.438371	0.6758668898	1.48
Sin(45°) = 0.707107	Sin(90°) = 0.453990	0.7071067812	1.41
		Average Refractive index	1.46

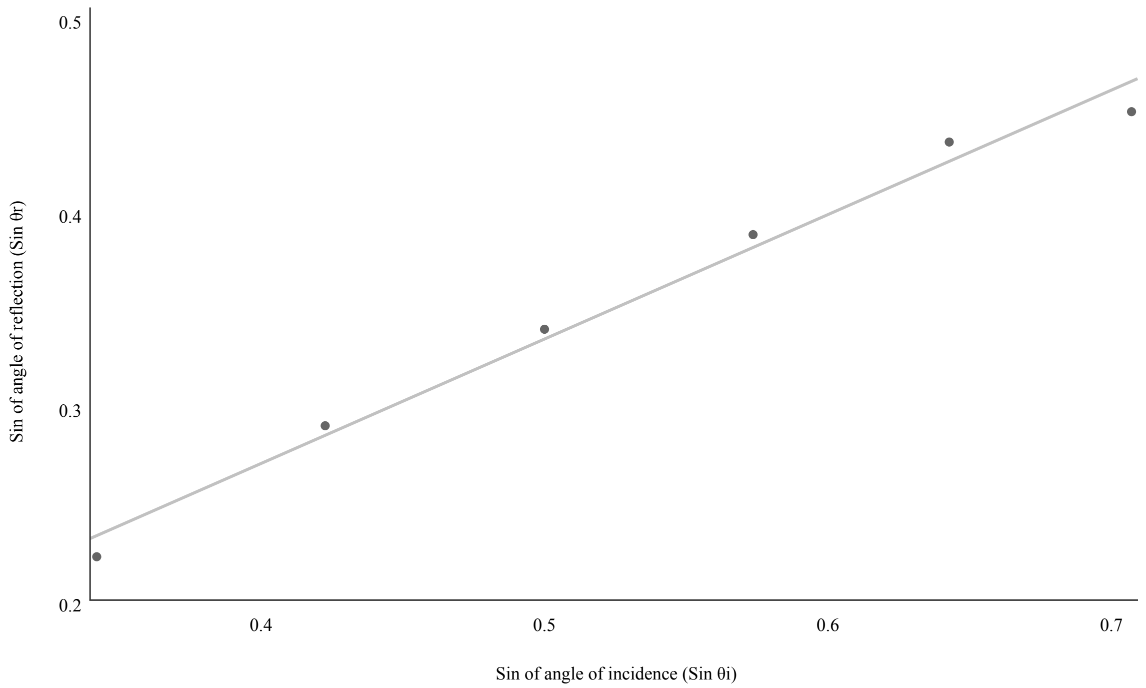

Figure 3.2: Angle of incidence Sin θ_{i} vs Angle of refraction Sin θ_{r}

Experiment 3: Measurement of the refractive index using apparent depth method.

Table 3: Experiment 3: Apparent Depth Perspex block

Mark on table (x_1)	Mark on block (x_2)	top of the block (x_3)
2.7	3.3	4.48
2.56	3.25	4.56
2.63	3.28	4.52

We can use the equation:

$$n = \frac{x_3 - x_1}{x_3 - x_2} \tag{7}$$

To calculate the refractive index of the perspex block like so:

$$\frac{4.48-2.7}{4.48-3.3} = 1.51,\ \frac{4.56-2.56}{4.56-3.25} = 1.53,\ \frac{4.52-2.63}{4.52-3.28} = 1.52$$

Average refractive index: 1.52, Standard Deviation: 0.008.

Table 4: Experiment 3: Apparent Depth Water

Mark on table (x_1)	Mark on bottom of water (x_2)	top of the water(x_3)
2.97	3.43	4.76
2.72	3.20	4.66
2.95	3.25	4.42

Reusing Eq. 7 we can figure out the refractive index of water:

$$\frac{4.76-2.97}{4.76-3.43} = 1.35,\ \frac{4.66-2.72}{4.66-3.20} = 1.33,\ \frac{4.42-2.85}{4.42-3.25} = 1.34$$

Average refractive index: 1.34, Standard Deviation: 0.008.

Table 5: Experiment 3: Apparent Depth Water

Mark on table (x_1)	Mark on bottom of glycerol (x_2)	top of the glycerol(x_3)
2.97	3.35	4.92
2.97	3.40	4.98
2.97	3.55	4.98

Once again we use Eq. 7 to work out the refractive index of glycerol:

$$\frac{4.92-2.97}{4.92-3.35} = 1.24,\ \frac{4.98-2.97}{4.98-3.40} = 1.27,\ \frac{4.98-2.97}{4.98-3.55} = 1.41$$

Average refractive index: 1.31, Standard Deviation: 0.081.

Above we calculated the refractive index (in table 1 and table 2) using Eq. 1 and Eq. 6 respectively.
but we can also determine the refractive index for our second set of results using Eq.3

While carrying out our second experiment with the Perspex block we discovered that at the angle $\theta_{\mathrm{i}} = 42°$ our beam of light produced by our laser no longer left the glass block and instead was internally refracted. We would describe this angle as our critical angle, thus allowing is to determine a refractive index.

Using Eq.3 we get:

$$sin(42°) = \frac{1}{n}$$

Our value for $\frac{1}{n} = 0.669$, inverting this we get

$$0.669^{-1} = 1.495$$

Which is entirely consistent with the values of n we have been getting thus far.

We can also work out the refractive index of the medium using the slope of our two plotted graphs. (Fig. 3.1 and Fig. 3.2)

The slope of Fig. 3.1 is 0.687,

$$0.687^{-1} = 1.46$$

The slope of Fig. 3.2 is 0.64,

$$0.64^{-1} = 1.56$$

4 Conclusion:

We determined the refractive index of the Perspex block was in the range of 1.46 - 1.56, with an average refractive index of: 1.491.

We also determined the refractive index of water to be on average 1.34 and the refractive index of glycerol on average to be 1.31, using the apparent depth method.