Determination of gravity by the free fall method

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2:20 P.M

1 Introduction:

In our day to day lives, gravity is without a doubt the most notable force, from keeping us on the ground, to causing things to fall towards the earth- gravity is prominently described as an anchoring force for most objects. It isn't surprising then- that a good method for calculating a magnitude for the force gravity has on objects is by measuring an object as it falls from a height. This is ultimately the theory behind the free fall method, if we can drop an object in a controlled environment where we know the height the object is being dropped from as well as the time it took the object to reach its resting place, then we can use these values to determine how strong this anchoring force really is.

Any object which is affected by gravity is described as being given an acceleration towards the earth by gravitational force, this is the reasoning behind the phrase 'Acceleration due to gravity' as such, the unit for gravity is ms^{-2}. The goal of our experiment will be to determine a value for this acceleration before using this value to help us find out other interesting information about the energy conversion taking place during the experiment.

Overall this experiment mostly deals with the motion which is occurring when the object is thrown upwards and then falls from a certain height, and the time taken to go from one height to another tells us about the motion of the object and the components that make it up. With this in mind it's not surprising that the most important equation in determining a value for g is an equation derived from motion:

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0 \tag{1}$$

Where g = gravitational acceleration, v_0 = initial velocity, t = time and y_0 = initial position.

This equation is perfect for the purpose of this experiment as we will already know the displacement of the object, due to the object being dropped straight down. This makes finding the unknown values extremely easy as plotting our time vs distance graph should give us a quadratic with co-efficience which map perfectly onto this equation.

When this happens finding our value for g will be as easy as multiplying the co-efficent of x^2 by 2, which is our value for acceleration (a). It is important to note that our value of acceleration will be expressed as a negative value, this is due to the force of gravity acting in opposition to how the ball initially wants to accelerate as it is being thrown upwards.

Not only will we get a value for gravity, we will also be given the value for initial velocity (u) with the co-efficient of x, as well as the initial height of the object which will be represented as a constant that isn't a part of the equation of motion.

2 Results & Data Analysis:

Table 1: Recorded values of free fall time from measured height

Times (s)	Positions (m)	Times (s)	Positions (m)	Times (s)	Positions (m)	Times (s)	Positions (m)	Times (s)	Positions (m)
0	0.460	0.210	1.200	0.410	1.474	0.610	1.370	0.810	0.882
0.010	0.540	0.220	1.215	0.420	1.478	0.620	1.355	0.820	0.845
0.020	0.588	0.230	1.237	0.430	1.490	0.630	1.339	0.830	0.820
0.030	0.608	0.240	1.258	0.440	1.483	0.640	1.350	0.840	0.760
0.040	0.640	0.250	1.279	0.450	1.484	0.650	1.300	0.850	0.738
0.050	0.670	0.260	1.298	0.460	1.480	0.660	1.284	0.860	0.699
0.060	0.727	0.270	1.340	0.470	1.483	0.670	1.264	0.870	0.660
0.070	0.764	0.280	1.334	0.480	1.495	0.680	1.250	0.880	0.610
0.080	0.790	0.290	1.351	0.490	1.479	0.690	1.221	0.890	0.578
0.090	0.837	0.300	1.366	0.500	1.475	0.700	1.198	0.900	0.536
0.100	0.890	0.310	1.400	0.510	1.478	0.710	1.174	0.910	0.493
0.110	0.906	0.320	1.394	0.520	1.480	0.720	1.149	0.920	0.446
0.120	0.920	0.330	1.435	0.530	1.450	0.730	1.123	0.930	0.403
0.130	0.971	0.340	1.430	0.540	1.460	0.740	1097	0.940	0.357
0.140	1.002	0.350	1.430	0.550	1.450	0.750	1.069	0.950	0.310
0.150	1.032	0.360	1.439	0.560	1.432	0.760	1.040	0.960	0.262
0.160	1.100	0.370	1.430	0.570	1.440	0.770	1.010	0.970	0.213
0.170	1.089	0.380	1.456	0.580	1.411	0.780	0.980	0.980	0.163
0.180	1.100	0.390	1.498	0.590	1.398	0.790	0.940	0.990	0.112
0.190	1.142	0.400	1.480	0.600	1.390	0.800	0.915	1.000	0.060
0.200	1.167	—	—	—	—	—	—	—	—

Using the LINEST function on google sheets we are able to determine that the uncertainty of the slope is: 0.127
and the uncertainty of the y-intercept is: 0.074

Using the LINEST function on google sheets we are able to determine that in the equation:

$$y(t) = a_1 t^2 + a_2 t + a_3 \tag{2}$$

Our values for $a_1 = -4.85$, $a_2 = 4.42$ and $a_3 = 0.48$

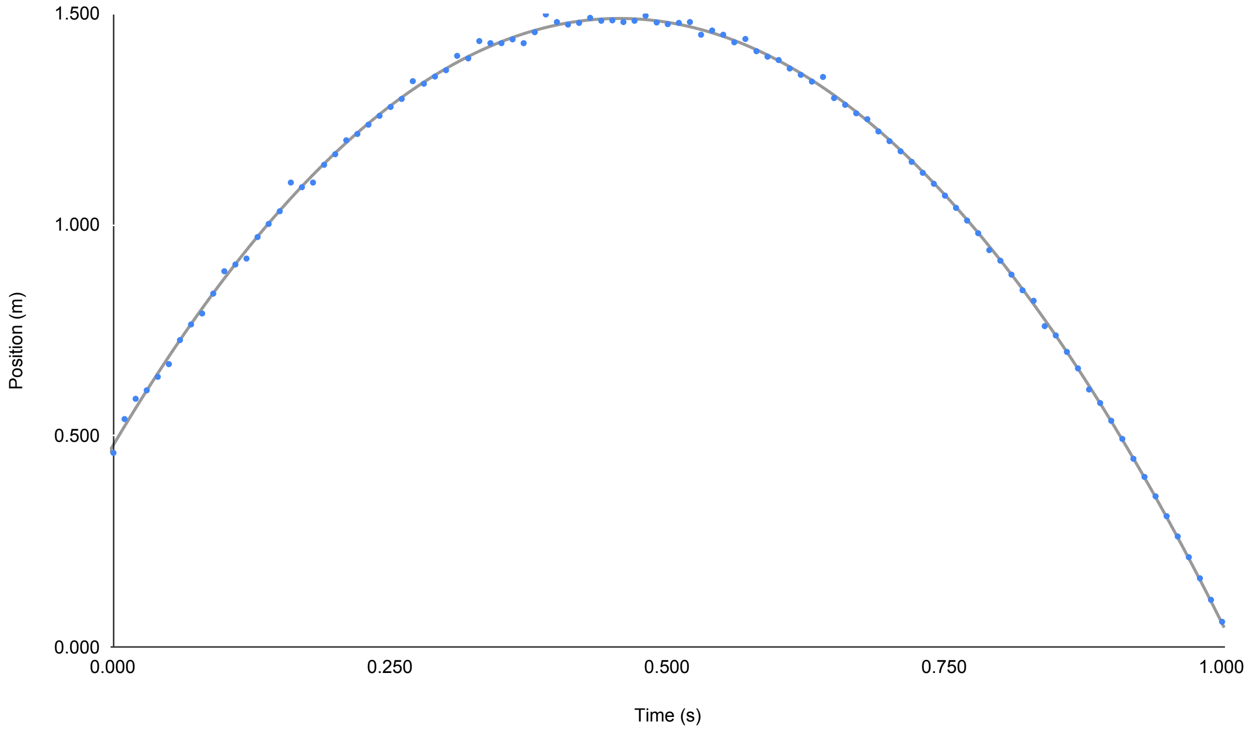

Figure 2.1: T(s) vs Position (m)

Our equation for motion is as follows:

$$\frac{1}{2}\,a_1\,t^2 + a_2\,t + a_3 \tag{3}$$

Subbing in our values:

$$-4.85t^2 + 4.42t + 0.48$$

We know that -4.85 is half the acceleration, therefore $a =$ -9.7

The acceleration in this case being gravity. So we have determined that our value for g is -9.7 ms^{-1}

The initial velocity of the object is presented by a_2, so we know that the initial velocity is 4.42 ms^{-1}

The initial height of the object is represented by a_3, known as the y-intercept of the equation, which is the position of the ball at 0 seconds. We know the initial height of the ball is 0.48m

We can differentiate out our equation of motion from:

$$-4.85t^2 + 4.42t + 0.48$$

to give us:

$$-9.7t + 4.42$$

We can substitute in our recorded measurements for time into t and get out values of the velocity of the ball at each point in time. Once we have these values we can graph them vs the time to get a Time vs Velocity graph. Our graph looks like this:

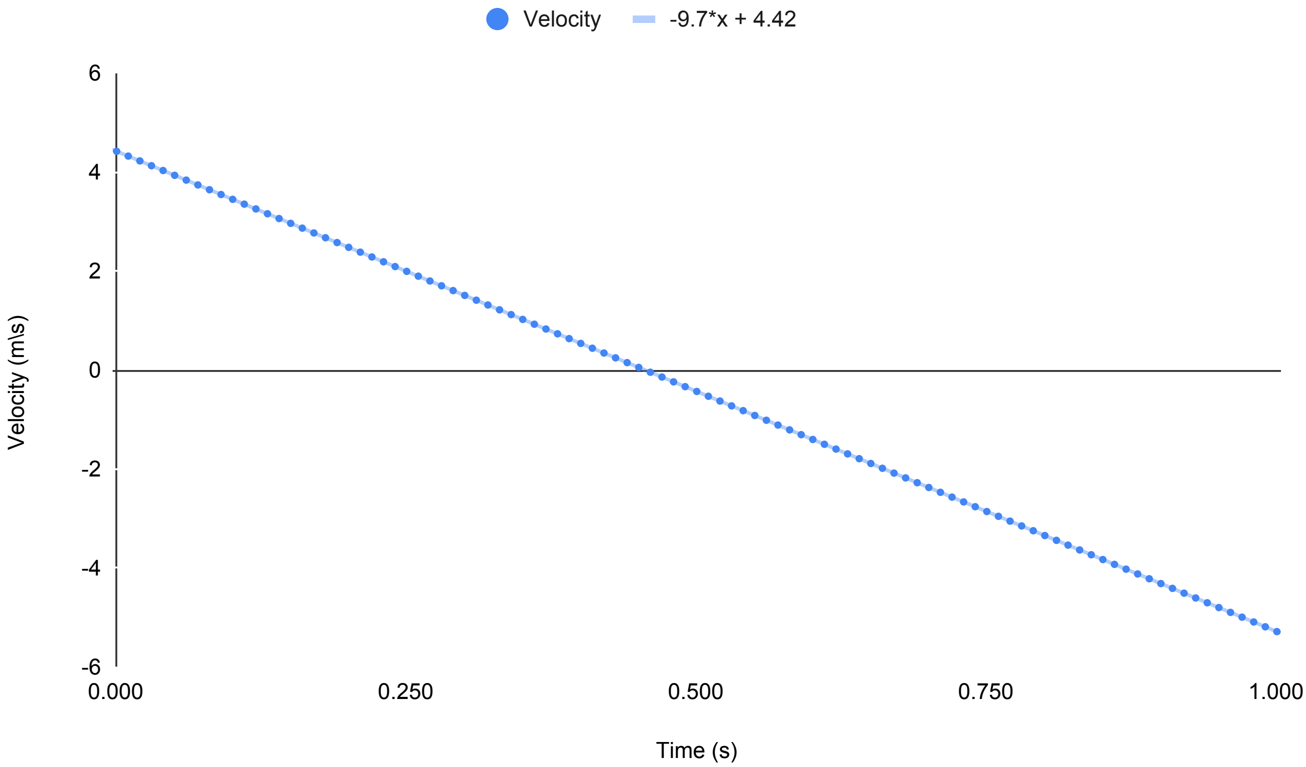

Figure 2.2: T(s) vs Velocity (ms^{-1})

The linear nature of this graph is what we expect to see, and the slope of the graph verifies that the acceleration is infact -9.7 ms^{-1}

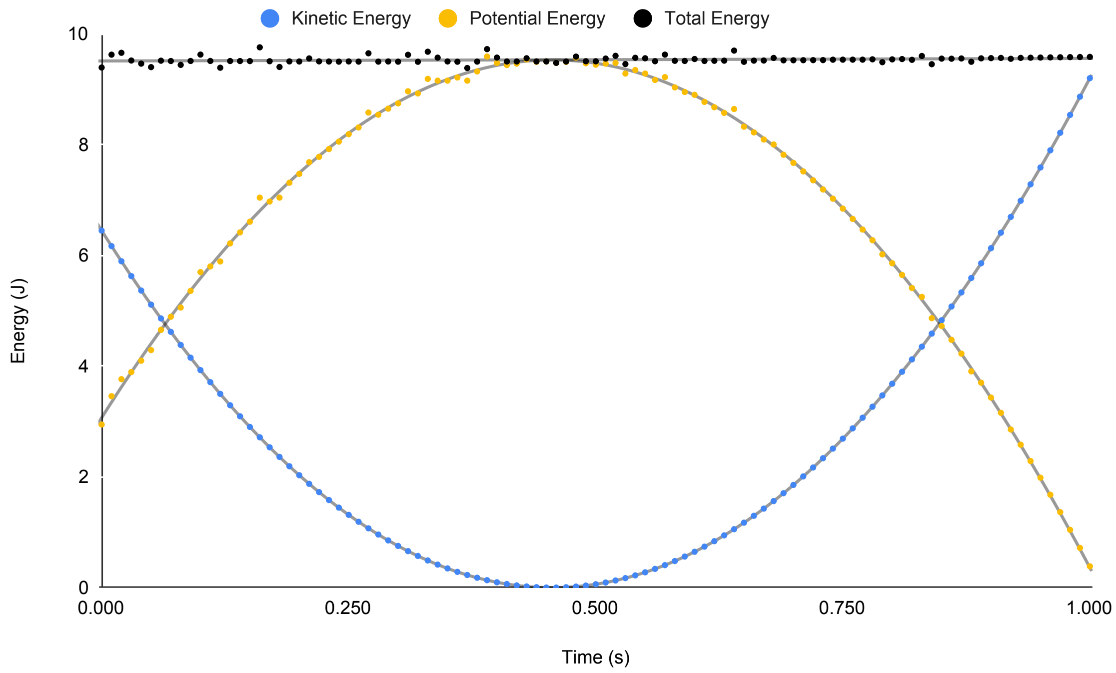

Figure 2.3: T(s) vs E_k, E_p and E_{total}(J)

This graph maintains the relationship we know exists between E_k and E_p, As the ball moves from a position of high potential energy and starts to move we can see the relationship carry out where the potential energy decreases and the kinetic energy increases proportionally, while also keeping the total energy constant, at any point on the graph we can add our Potential energy and Kinetic energy together and always arrive at our value for our total energy, this shows that energy isn't being created or destroyed but is changing from one form to another, thus the conservation of energy is sustained.

We can now use the standard error gotten from the LINEST function, our error for a_1 was 0.01, our a_2 was 0.01 and a_3 was 0.003
Our final values are: $g = 9.7 \pm 0.02,\ v_0 = 4.42 \pm 0.01,\ y_0 = 0.48 \pm 0.003$.

We can now compare our calculated value for gravity: 9.7 ms^{-2} vs our known value for gravity 9.81 ms^{-2}

$$\frac{|Theoretical\ Value - Experimental\ Value|}{Theoretical\ Value} \times 100 \tag{4}$$

$$\frac{|\ 9.81 - 9.7\ |}{9.81} \times 100 = 1.12\%$$

3 Conclusion:

We determined an experimental value for g of 9.7 which as a difference of 1.12% of what we would expect to see. We were also able to visually represent the conservation of energy by graphing our Kinetic Energy vs our Potential Energy vs our Total Energy of the system.

Table 2: Full Table of Data used/calculated

Times (s)	Positions (m)	Velocity (ms^{-1})	Kinetic Energy (J)	Potential Energy (J)	Total Energy (J)
0	0.460	4.420	6.447	2.945	9.392
0.010	0.540	4.323	6.167	3.457	9.624
0.020	0.588	4.226	5.893	3.764	9.658
0.030	0.608	4.129	5.626	3.892	9.518
0.040	0.640	4.032	5.365	4.097	9.462
0.050	0.670	3.935	5.110	4.289	9.399
0.060	0.727	3.838	4.861	4.654	9.515
0.070	0.764	3.741	4.618	4.891	9.510
0.080	0.790	3.644	4.382	5.058	9.440
0.090	0.837	3.547	4.152	5.358	9.510
0.100	0.890	3.450	3.928	5.698	9.626
0.110	0.906	3.353	3.710	5.800	9.510
0.120	0.920	3.256	3.499	5.890	9.388
0.130	0.971	3.159	3.293	6.216	9.510
0.140	1.002	3.062	3.094	6.415	9.509
0.150	1.032	2.065	2.901	6.607	9.508
0.160	1.100	2.965	2.714	7.042	9.757
0.170	1.089	2.868	2.534	6.972	9.506
0.180	1.100	2.771	2.360	7.042	9.402
0.190	1.142	2.674	2.192	7.311	9.503
0.200	1.167	2.577	2.030	7.471	9.501
0.210	1.200	2.480	1.874	7.682	9.556
0.220	1.215	2.383	1.725	7.778	9.503
0.230	1.237	2.286	1.581	7.919	9.501
0.240	1.258	2.189	1.444	8.054	9.498
0.250	1.279	2.092	1.313	8.188	9.502
0.260	1.298	1.995	1.189	8.310	9.499
0.270	1.340	1.898	1.070	8.579	9.649
0.280	1.334	1.801	0.958	8.540	9.498
0.290	1.351	1.704	0.852	8.649	9.501
0.300	1.366	1.607	0.752	8.745	9.498
0.310	1.400	1.510	0.659	8.963	9.622
0.320	1.394	1.413	0.572	8.924	9.496
0.330	1.435	1.316	0.490	9.187	9.677
0.340	1.430	1.219	0.415	9.155	9.570
0.350	1.430	1.122	0.347	9.155	9.502
0.360	1.439	1.025	0.284	9.212	9.497
0.370	1.430	0.928	0.228	9.155	9.383
0.380	1.456	0.831	0.178	9.321	9.499
0.390	1.498	0.734	0.134	9.590	9.724
0.400	1.480	0.637	0.096	9.475	9.571
0.410	1.474	0.540	0.065	9.437	9.501
0.420	1.478	0.443	0.040	9.462	9.502
0.430	1.490	0.346	0.020	9.539	9.559
0.440	1.483	0.249	0.008	9.494	9.502
0.450	1.484	0.152	0.001	9.501	9.502
0.460	1.480	0.055	0.001	9.475	9.476
0.470	1.483	-0.042	0.006	9.494	9.501
0.480	1.495	-0.139	0.018	9.571	9.504
0.490	1.479	-0.236	0.037	9.469	9.554
0.500	1.475	-0.333	0.061	9.443	9.603
0.510	1.478	-0.430	0.092	9.462	9.454
0.520	1.480	-0.527	0.128	9.475	9.568
0.530	1.450	-0.624	0.172	9.283	9.559
0.540	1.460	-0.721	0.221	9.347	9.506
0.550	1.450	-0.818	0.276	9.283	9.625
0.560	1.432	-0.915	0.338	9.168	9.513
0.570	1.440	-1.012	0.406	9.219	9.510
0.580	1.411	-1.109	0.480	9.033	9.546
0.590	1.398	-1.206	0.560	8.950	9.510
0.600	1.390	-1.303	0.647	8.899	9.513
0.610	1.370	-1.400	0.740	8.771	9.516
0.620	1.355	-1.497	0.838	8.675	9.698
0.630	1.339	-1.594	0.944	8.572	9.495
0.640	1.350	-1.691	1.055	8.643	9.517
0.650	1.300	-1.788	1.173	8.323	9.518
0.660	1.284	-1.885	1.296	8.220	9.565
0.670	1.264	-1.982	1.426	8.092	9.523
0.680	1.250	-2.079	1.563	8.003	9.523
0.690	1.221	-2.176	1.705	7.817	9.524
0.700	1.198	-2.273	1.854	7.670	9.525
0.710	1.174	-2.370	2.008	7.516	9.526
0.720	1.149	-2.467	2.169	7.356	9.533
0.730	1.123	-2.564	2.337	7.189	9.534
0.740	1.097	-2.661	2.510	7.023	9.534
0.750	1.069	-2.758	2.690	6.844	9.534
0.760	1.040	-2.855	2.876	6.658	9.540
0.770	1.010	-2.952	3.068	6.466	9.489
0.780	0.980	-3.049	3.266	6.274	9.539
0.790	0.940	-3.146	3.471	6.018	9.545
0.800	0.915	-3.243	3.681	5.858	9.531
0.810	0.882	-3.340	3.900	5.647	9.600
0.820	0.845	-3.437	4.121	5.410	9.452
0.830	0.820	-3.534	4.351	5.250	9.553
0.840	0.760	-3.631	4.586	4.866	9.551
0.850	0.738	-3.728	4.828	4.725	9.556
0.860	0.699	-3.825	5.076	4.475	9.496
0.870	0.660	-3.922	5.330	4.225	9.558
0.880	0.610	-4.019	5.591	3.905	9.562
0.890	0.578	-4.116	5.857	3.700	9.565
0.900	0.536	-4.213	6.130	3.431	9.550
0.910	0.493	-4.310	6.409	3.156	9.566
0.920	0.446	-4.407	6.694	2.855	9.569
0.930	0.403	-4.504	6.986	2.580	9.572
0.940	0.357	-4.601	7.283	2.286	9.574
0.950	0.310	-4.698	7.587	1.985	9.577
0.960	0.262	-4.795	7.897	1.677	9.580
0.970	0.213	-4.892	8.214	1.364	9.582
0.980	0.163	-4.989	8.536	1.044	9.584
0.990	0.112	-5.086	8.865	0.717	9.582
1.000	0.060	-5.183	9.200	0.384	9.584