Lab Report

Black-Body Radiation via Phase Sensitive Detection

Report by: **Jamie Somers**
Lab Partner: Adam Morrissey
Grader: Bert Ellingboe

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Abstract

The purpose of this experiment was to determine the true resistance of a tungsten filament without the resistance added by a circuit, the verification of both Stefan's law and Wien's displacement law and to find the cooling down rate of a tungsten filament all using a Lock-in Amplifier (LIA). This was done by setting up the tungsten filament bulb attached to a circuit to vary the voltage and current and using lenses separated by a chopper to focus the tungsten bulbs light onto a monochromator with a detector who's output is sent to an LIA along with a reference signal from the chopper to isolate the signal of the tungsten power. The cooling down rate of the tungsten filament was not determined in the scope of this lab report however the resistance of the filament was found to be $0.525 \pm 0.005\Omega$ which is consistent with the scientific literature. It appears from the results obtained in this experiment that both Stefan's law and Wien's law are verifiably true. It should be noted that Wien's Law did not remain true for wavelengths below 800nm and more measurements at wavelengths below this limit should be taken.

1 Introduction

According to the National Council on Radiation Protection and Measurements (NCRP), the average annual radiation dose per person in the U.S. is 6.2 millisieverts (620 millirem)[1]. It is no surprise then, that one of the biggest and most consistent forms of interference in physics lab equipment is background radiation. Any detector that is designed to detect electromagnetic waves or sensitive enough to pick up small vibrations will experience electrical noise which will affect the readings. Although there is no way to stop background radiation from occurring naturally, scientists have invented machines and techniques to reduce the affect that background radiation and noise have on the signals they are trying to identify in the lab.

One such machine which can help reduce the affect noise can have on trying to isolate a weak signal is a Lock-In Amplifier (LIA). Lock-In Amplifier's using a bandwidth limiting technique wherein the Lock-In Amplifier requires a reference signal of what signal it is attempting to isolate for and will average the multiple signals being detected and as a result the noise will be averaged to zero as it hopefully doesn't share and signal characteristics with the reference signal provided.

This experiment involves the use of a Lock-In Amplifier in collaboration with a miniature monochromator in an effort to isolate the signal coming from a chopped electromagnetic wave coming from a tungsten filament bulb at different temperatures and wavelengths. This allows us to explore the relationship that exists between Temperature and Wavelength and how Temperature scales with Power. These relationships are formally known as Wein's Law and Stefan's Law respectively and are both described under Planck's Radiation Law.

2 Background and Theory

When determining the relationship between Wavelength and Temperature / Power and Temperature its important to first have an accurate value for the temperature. In this experiment a tungsten filament bulb is used which has a known relationship with resistance that allows us to calculate its temperature, the relationship is as follows:

$$T = T_{300}\left(\left(\frac{R}{R_{300}}\right)^{0.82984}\right) \tag{1}$$

Where T is the temperature of the filament, T_{300} is the temperature of the room (approximately 300K), R is the resistance of the filament bulb at temperature T and R_{300} is the resistance of the filament bulb at room temperature.

It is equally as important to have an accurate value for the resistance which exists inside of the filament bulb circuit. Unfortunately the resistance measured in the lab using the voltage and current of the circuit is not a direct value of the filament bulbs resistance. This is due to all of the extra circuitry such as the wires, connectors and socket resistance which impede the electrons ability to flow within the circuit. We can more accurately define the resistance in the circuit as:

$$R_{\text{total}} = R_{\text{wires}} + R_{\text{filament}} \tag{2}$$

Where R_{total} is the resistance seen by the power supply, R_{wires} accounts for all of the added resistances not caused by the filament bulb and R_{filament} is the true resistance of the filament bulb.

In order to calculate the full bulb circuit resistance we will use the following equation:

$$R_{fc} = 0.5(V_{fc}/V_{0.5}) \text{ ohms} \tag{3}$$

Where R_{fc} is the resistance of the full circuit, V_{fc} is the voltage measured across the full circuit and $V_{0.5}$ is the voltage measured across a 0.5 Ω resistor.

We will verify two equations, the first law Stefan's law describes the relationship between the radiant emittance of a black body and the temperature of the black body. The law is as follows:

$$j^* = A\sigma T^4 \tag{4}$$

Where j^* is the black-body radiant emittance, A is the surface area of the emitter, σ is the Stefan–Boltzmann constant and T^4 is the temperature in kelvin to the fourth power.

The other law we will be attempting to verify is known as Wien's displacement law and its equation is as follows:

$$\lambda_{max} = \frac{b}{T} \tag{5}$$

Where λ_{max} is the wavelength peak, b is the constant of proportionality ($2.897771955... \times 10^{-3} m \cdot K$) and T is the absolute temperature in kelvin.

3 Experimental Design and Procedure

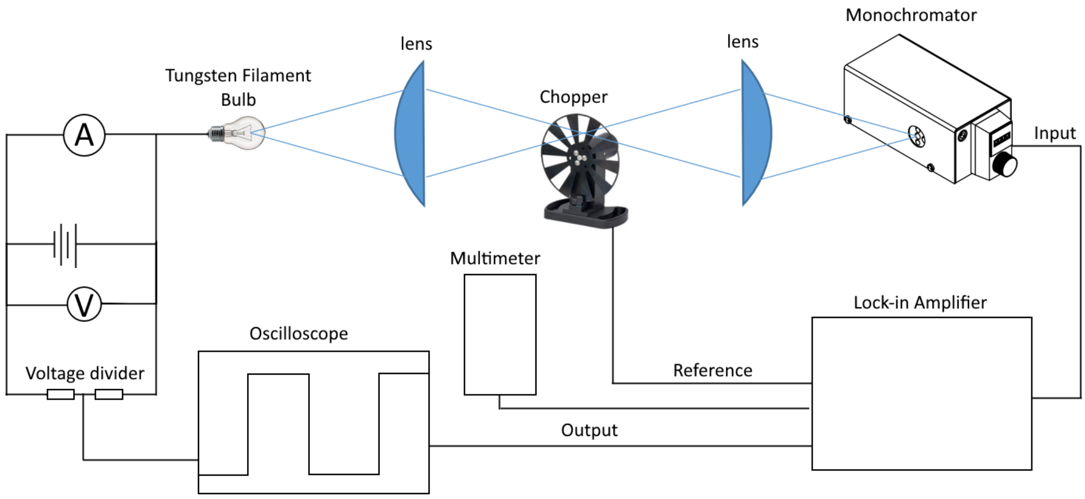

Figure 1: Diagram of experimental set-up for Black-body Radiation Experiment

This experimental set-up included a Tungsten filament bulb connected to a circuit containing an ammeter and voltmeter. Two lenses for focusing the tungsten ray of light through an automatic spinning chopper and onto the split of a monochromator. A Lock-In Amplifier (LIA) which increases the signal measured by the detector on the output of the monochromator using a signal from the chopper as a reference, and an oscilloscope which gives a real time graphical representation of the signal gotten from the detector that is cleaned up by the LIA.

The first part of the experiment involved finding the resistance of the cold tungsten filament at room temperature. This was first done by using a handheld multimeter to measure the resistance directly, but was improved upon by comparing the signal on an oscilloscope for a 5Ω resistance and comparing it with the signal for the filament bulb using a 5V p-p square wave signal as a reference. The tungsten filament bulb was replaced with a shorted bulb which should have zero resistance across it, giving us the resistance for the rest of the circuit. We then use Equation 2 and 3 to improve the accuracy of this calculated resistance.

The experimental apparatus was set up as shown in the diagram above. The monochromator slit was adjusted so light of all wavelengths pass through in "zero order" and the slit distance was recorded in mm. The chopper was set to 131 Hz and its reference signal was connected to the reference input of the LIA and a square wave was observed from the oscilloscope. The phase setting of the LIA was adjusted to ensure the output signal is maximised and the chopper frequency was changed to observe what frequencies gave a clean signal which could be analysed accurately.

The tungsten filament was attached to two multimeters, one set up as an ammeter is series and one set up as a voltmeter in parallel. This allowed us to change the voltage received by the tungsten filament and and measure both the voltage and current. Using Ohm's Law $V = IR$ to find the resistance one can then use Equ. 1 to find the temperature of the tungsten filament at different voltages (12 different power levels). We can then verify Stefan's Law for a black body as described in Equ. 4.

The next part of the experiment involves setting the filament to glow a dull red colour and record its current and voltage. The values of the detector, monochromator and LIA were all recorded for a range of wavelengths found by changing the slit of the monochromator. Increasing the voltage of the filament we get a wide variety of wavelengths at different temperatures of the filament, this cant be used to verify Wien's law Eq.5 which describes that the relationship between wavelength peak and temperature is inversely proportional.

The cooling down rate of the tungsten filament can be found by connecting the tungsten filament to the voltmeter and increasing the voltage to 12 volts, after 10 seconds the filament should be completely heated and the circuit can be disconnected and moved to the voltage divider circuit. Once the circuit is connected the output of the LIA can be recorded every 5 seconds, the amount the filament cools down by will decrease over time so the recording intervals can be increased. Then by plotting V_{f*} vs Time you can find the cooling down rate from the slope.

4 Results

4.1 Cold Tungsten Filament Resistance Measurement

We can do a very simple calculation with these two measured values and determine what resistance is contributed by the system. Once we determine the uncertainty of our values we can see exactly why getting the resistance through an LIA is a lot more accurate.

$$\text{Cheap Multimeter: } 1.2 \pm 0.1\Omega - 0.7 \pm 0.1\Omega = 0.5 \pm 0.14\Omega$$

Table 1: Filament bulb resistance measurements

Filament Bulb off	R (Ω) $\pm$ 0.1Ω	Dummy Bulb	R (Ω) $\pm$ 0.1Ω
Cheap Multimeter	1.2	Cheap Multimeter	0.7
Bench multimeter	1.311	Bench multimeter	0.622

$$\text{Digital Multimeter: } 1.311 \pm 0.1\Omega - 0.622 \pm 0.1\Omega = 0.689 \pm 0.14\Omega$$

Error Analysis:

$$\Delta z = \sqrt{(0.1)^2 + (0.1)^2} = 0.14$$

The resistances calculated have an error of 28% and 20% respectively.This is an extremely high level of uncertainty for such a relatively small measurement and confirms our need for a more accurate method of measuring the resistance of the filament such as the LIA method used below.

Table 2: Filament bulb resistance measurements

Filament Bulb off	Voltage (V) $\pm$0.0005	Dummy Bulb	Voltage (V) $\pm$0.0005
4K7 Ω	0.672	4K7 Ω	0.364
4K7 Ω + 5 Ω	0.293	4K7 Ω + 5 Ω	0.292

We can now carry out our calculations on these new measurements:

$$V_{\text{full}} = V_{\text{full circuit}} - V_{\text{short circuit}}$$
$$V_{\text{full}} = 0.672 - 0.364 = 0.308V$$
$$\frac{0.308V}{0.293V} = 1.05$$
$$\frac{V_{0.5\Omega}}{R_{0.5\Omega}} = \frac{V_{\text{filament only}}}{R_{\text{filament only}}}$$
$$R_{\text{filament only}} = R_{0.5\Omega}\left(\frac{V_{\text{filament only}}}{V_{0.5\Omega}}\right)$$
$$R_{\text{filament only}} = 0.5(1.05) = 0.525\Omega$$

Using the LIA we have measured the resistance of the tungsten filament to be 0.525 $\pm$ 0.005Ω

Error Analysis:

$$\Delta z = \left|\frac{0.308V}{0.293V}\right| \sqrt{\left(\frac{0.0005}{0.308}\right)^2 + \left(\frac{0.0005}{0.293}\right)^2} = 2.476 \times 10^{-3}$$

$$\Delta z = |(0.5)(1.05)| \sqrt{\left(\frac{0.005}{0.5}\right)^2 + \left(\frac{2.476 \times 10^{-3}}{1.05}\right)^2} = 5.394 \times 10^{-3}$$

4.2 Total Power Radiated By Blackbody Sources

We can use ohms law $V = IR$ and Eq. 1 to find the temperature of the filament at different power levels. We can then use these values verify if Stefan's law holds. The temperature of the room is assumed to be $300K$ and the resistance of the filament at room temperature is $R_{300} = 0.689 \pm 0.14\Omega$

$$R = \frac{V}{I} = \frac{3.61}{0.931} = 3.88 \pm 0.059\Omega$$

$$T = T_{300}\left(\left(\frac{R}{R_{300}}\right)^{0.82984}\right) = (300K)\left(\left(\frac{3.88\Omega}{0.689\Omega}\right)^{0.82984}\right) = 1258.95K$$

Table 3: List of 12 different power levels (Voltage and Current measurements)

V (V)	I (mA)	R (Ω)	T (K)	T^4 (K^4)
1.69 ± 0.009	669 ± 13.97	2.53 ± 0.05	881.76 ± 0.75	$6 \times 10^{11} \pm 2 \times 10^{9}$
2 ± 0.004	713 ± 10.08	2.81 ± 0.04	961.82 ± 0.83	$8.56 \times 10^{11} \pm 2.9 \times 10^{9}$
2.49 ± 0.006	787 ± 11.805	3.16 ± 0.05	1062.87 ± 0.93	$1.28 \times 10^{12} \pm 4.4 \times 10^{10}$
3 ± 0.008	855 ± 12.825	3.51 ± 0.05	1158.15 ± 1.04	$1.80 \times 10^{12} \pm 3 \times 10^{10}$
3.5 ± 0.009	917 ± 13.755	3.82 ± 0.06	1241.91 ± 1.13	$2.38 \times 10^{12} \pm 5.2 \times 10^{10}$
3.61 ± 0.010	931 ± 14.655	3.88 ± 0.06	1258.95 ± 1.15	$2.51 \times 10^{12} \pm 5 \times 10^{10}$
3.99 ± 0.011	977 ± 15.45	4.08 ± 0.07	1313.62 ± 1.21	$2.98 \times 10^{12} \pm 6 \times 10^{10}$
4.45 ± 0.014	1030 ± 17.13	4.32 ± 0.07	1376.43 ± 1.28	$3.59 \times 10^{12} \pm 7.7 \times 10^{10}$
5.48 ± 0.015	1142 ± 17.865	4.80 ± 0.08	1501.72 ± 1.42	$5.09 \times 10^{12} \pm 5.09 \times 10^{10}$
5.96 ± 0.017	1191 ± 19.08	5.00 ± 0.08	1554.92 ± 1.48	$5.85 \times 10^{12} \pm 8 \times 10^{10}$
6 ± 0.015	1194 ± 17.91	5.03 ± 0.08	1560.32 ± 1.49	$5.92 \times 10^{12} \pm 8.1 \times 10^{10}$
6.78 ± 0.005	1272 ± 10.695	5.33 ± 0.04	1638.53 ± 1.57	$7.21 \times 10^{12} \pm 5 \times 10^{10}$

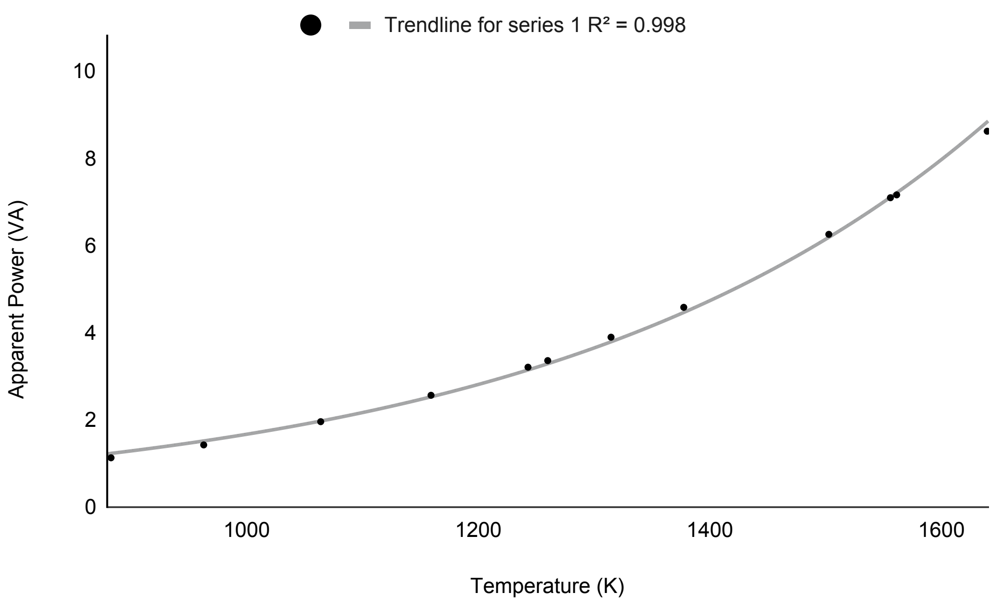

Figure 2: Graph of Apparent Power as a function of Temperature T.

We can see from Fig 1. a strong exponential trend in the graph. This somewhat verifies Stefan's law in the sense that there is some type of exponential relationship between Apparent power of the black body and power, however we can explore this further by checking the relationship between apparent power and Temperature T^4 specifically.

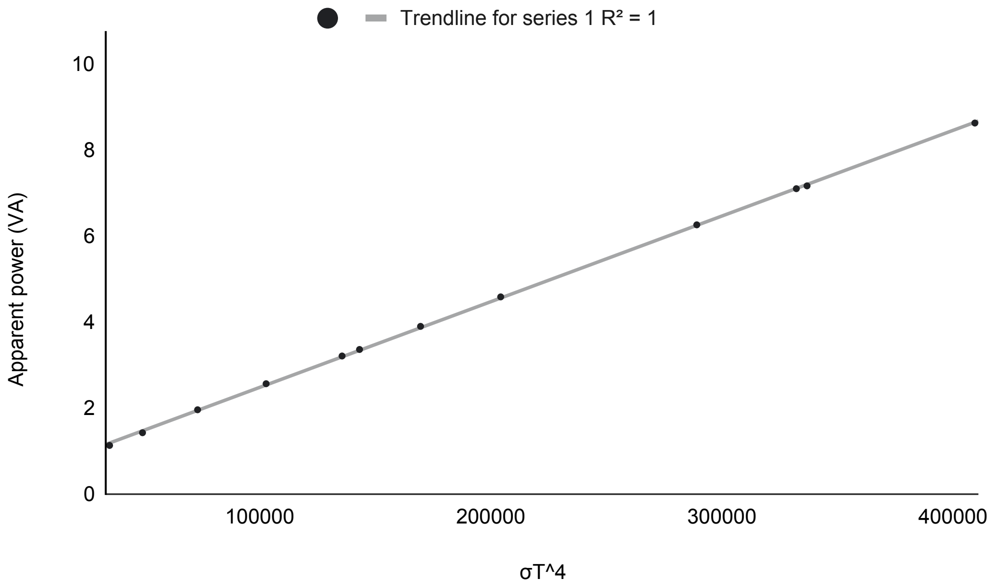

Figure 3: Graph of Apparent Power as a function of σT^4.

When we graph the relationship that exists between apparent power and the Temperature to the fourth power multiplied by the Stefan–Boltzmann constant we get a perfect linear relationship. This graph verifies Stefan's law which states that these two values have an exactly linear relationship.

Error Analysis:
R:

$$\Delta z = \left|\frac{1.69V}{0.669V}\right| \sqrt{\left(\frac{0.009}{1.69}\right)^2 + \left(\frac{0.01397}{0.669}\right)^2} = 0.05$$

T:

$$\Delta z = \left|\frac{2.53\Omega}{0.689\Omega}\right| \sqrt{\left(\frac{0.05}{2.53}\right)^2 + \left(\frac{0.14}{0.689}\right)^2} = 0.75$$

T_4:

$$\Delta z = |4|\, 881.76^{4-1}(0.75) = 2 \times 10^9$$

4.3 Spectral Radiance of a Filament Source

Table 4: Wavelength and LIA voltages at 5.01 V

Wavelength (nm)	LIA Reading (V)	Grating Efficiency (%)
402	3.75	-
658	2.2	-
866	8.3	10
1066	2.225	40
1298	0.983	62
1466	0.469	83

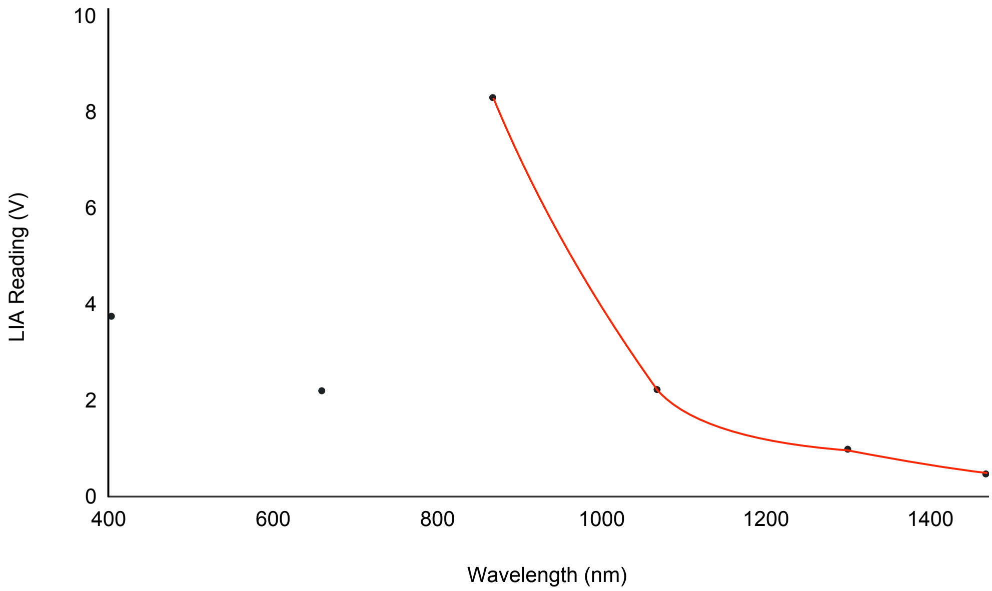

Figure 4: Graph of LIA Reading as a function of Wavelength. (5.01V)

We only have the grating efficiency for the last four measured wavelengths from the grating efficiency graph so we can discard the first two readings.

Table 5: Wavelength and LIA voltages at 7.5 V

Wavelength (nm)	LIA Reading (V)	Grating Efficiency (%)
990	2.304	10
1128	1.08	40
1302	0.94	62
1500	0.988	80
1728	0.72	75
1924	0.508	62

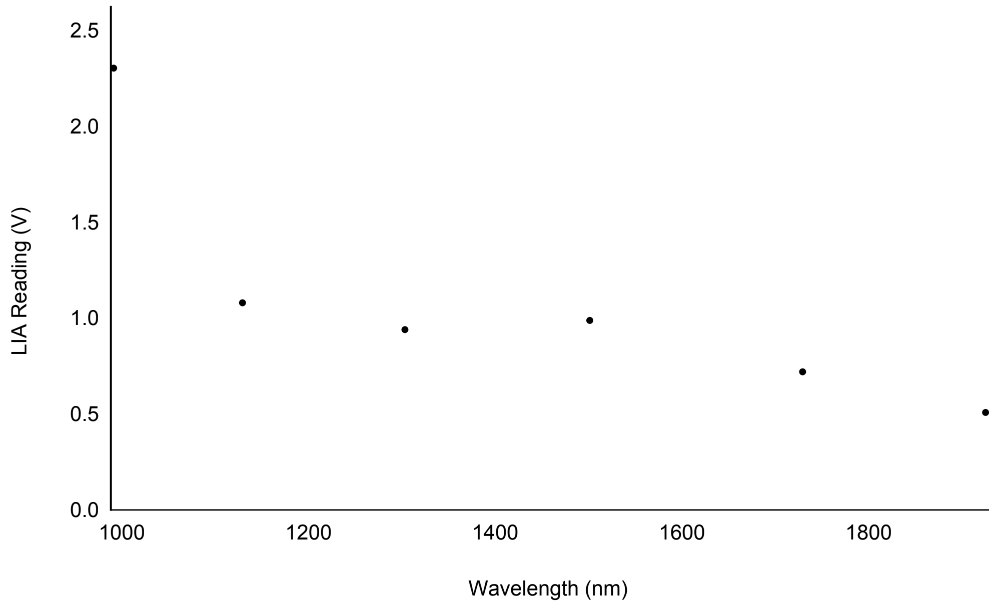

Figure 5: Graph of LIA Reading as a function of Wavelength. (7.5V)

Table 6: Wavelength and LIA voltages at 8.99 V

Wavelength (nm)	LIA Reading (V)	Grating Efficiency (%)
840	1.783	10
1080	0.637	40
1250	0.357	62
1536	0.36	80
1760	0.274	75
1890	0.203	65

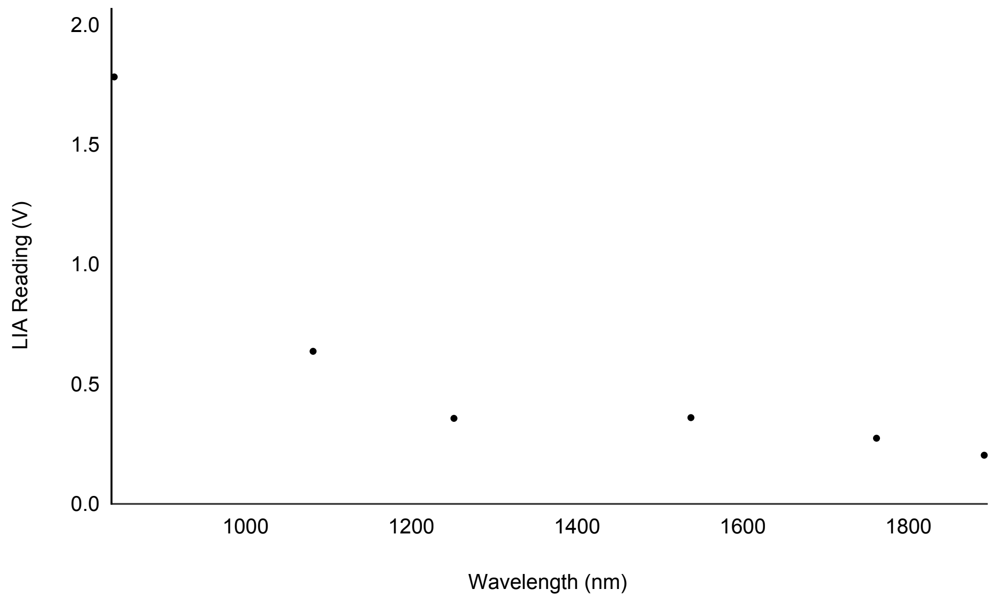

Figure 6: Graph of LIA Reading as a function of Wavelength. (8.99V)

These graphs, excluding the data points we don't have grating efficiency values for all follow a trend where as the wavelength increases, the value of the reading from the LIA decreases. This relationship verifies Wien's law which describes that the relationship between wavelength and temperature in inversely proportional. When the LIA reading is high (which represents the temperature of the filament bulb) the wavelength values are low, and when the LIA readings are low (the temperature of the filament bulb is low) the wavelengths are higher.

4.4 USE OF LOCK-IN AMPLIFIER

Table 7: List of useful chopper frequencies

Frequency (Hz)	Observation
0	No signal, not useful
5	Signal blinking, not useful
20	Still blinking, still not useful
30	Square waves starts to form, curves not defined. Not useful
40	Square wave is fully formed, still not defined, Not useful
60	Square wave formed, prominent curves and noise is visible
< 60	No more improvements in signal, more square waves formed.

We can determine from testing the chopper on frequencies between 0 - 400Hz that 60 to 200 Hz is the ideal frequency for a useful signal. Anything less than 60 Hz results in a less defined wave which is difficult to analyse and anything greater than 200 Hz results in too many square waves appearing on the oscilloscope to be distinguishable.

We also measured the LIA signal output (V) against the change in chopper frequencies.During the measurements the voltage was set to 6.47V and the current was at 1240mA, the measurements are included on the table below.

Table 8: List of LIA signal output (V) and chopper frequency (Hz) measurements

Frequency (Hz)	LIA Signal Output (V)	Time constant (s)
40	1.5985	3
80	1.658	3
120	1.651	3
160	1.623	3
200	1.583	3
240	1.538	3
280	1.491	3
320	1.449	3
360	1.402	3
400	1.359	3

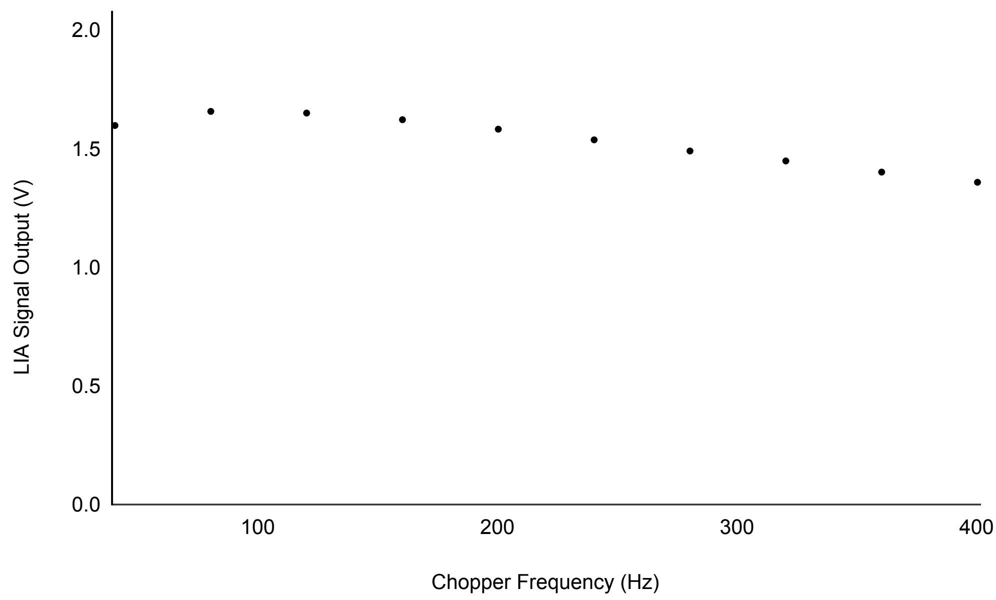

Figure 7: Graph of LIA Signal Output (V) as a function of Chopper Frequency (Hz)

5 Discussion and Conclusion

When working with equipment designed to detect radiation there is always going to be the issue of background radiation which exists all around us in our everyday life being detected instead of the subject we are trying to measure. Despite every measure implemented to reduce the affect of background radiation on the detector signal including the use of a lock-in amplifier to reduce noise and isolate the signal desired based on a reference signal there was still some residual noise affecting the signal shape.

Despite the affect that background radiation had on the experiment we were able to achieve relatively clean graphs with extremely strong relationships to the laws which we were trying to verify. We were able to verify that the Apparent Power of the tungsten filament increasing does increase the temperature by a factor of 4 (Stefan's law). We also observed from our graphs that Temperature of the tungsten filament does have an inverse relationship with the wavelength of light detected by the monochromator (Wien's law).

We concluded that the optimal frequency for the chopper is between the range of 60 Hz and 200 Hz as the signal produced is a near perfect square wave with extremely distinct curves from the background noise and there aren't too many square waves that observing the characteristics of the signal becomes difficult. The increase in chopper frequency appears to increase the LIA Output Signal up until 80Hz after which an increase in the frequency appears to decrease the LIA Output Signal.

The resistance of the tungsten filament was calculated to be $0.525 \pm 0.005\Omega$ which is far more accurate than the one significant figure with an error of 28% value we obtained from a cheap multimeter. This calculated value is also consistent with the book value for a tungsten filament which is typically between 0.2 Ω and 0.5 Ω

The cooling down rate of the tungsten filament was not carried out during the course of this experiment however if this experiment was to be replicated the cooling down rate could be determined using the method set out in the procedure section of this lab report.

6 References

[1] United States Environmental Protection Agency ”Radiation Sources and Doses — US EPA” https://www.epa.gov/radiation/radiation-sources-and-doses Apr. 9, 2021