PS157 Home Experiment: A Falling Hollow Paper Ball

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Tuesday 31^{st} March, 2020
10:00 A.M - 1:00 P.M

1 Introduction:

When an experiment is carried out in the lab, often times certain factors aren't accounted for due to them being considered negligible and thus the assumption can be made that it will not impact the overall experiment. This generalisation of complex systems down to simple models using assumptions has become so well known that the term 'Spherical cow'[1] was coined, which is a humorous metaphor which uses the analogy of a real life cow being substituted with a spherical cow that exists inside a vacuum which- in theory, would make calculations easier.

In the case of the Bouncy Ball Experiment, the air resistance acting opposite the balls motion known as 'Drag' was considered to be negligible for the duration of the experiment as well as within our calculations, however in reality the drag force was still playing a role within the actual real world experiment being done. In this experiment a hollow paper ball will be used in an effort to show that air resistance on a falling object can't always be considered negligible.

When we consider air to be a fluid, we can imagine that an object moving through air will have an affect on the momentum of the air. If we consider Newton's Third Law: For every action, there is an equal and opposite re-action, we can infer that the air is also affecting the object acting against its primary velocity. The magnitude of that force is defined by:

$$F_D = \tfrac{1}{2}C_D A\rho v^2 \tag{1}$$

where F_D is the force of drag, C_D is the drag coefficient, A is the cross sectional area of the object, ρ is the density of the fluid and v is the speed of the falling object.
When using Newtons Second Law: $F = ma$ while accounting for drag force you get:

$$ma = mg - F_D \tag{2}$$

where m is the mass of the object, a is the acceleration of the object, g is the acceleration due to gravity and F_D is the previously mentioned force of drag.
As the body accelerates, F_D will continue to increase. Once $F_D = mg$ the equation will result in $ma = 0$ this is because the body will no longer be acceleration and **"Terminal Velocity"** will have been achieved. The equation to determine this velocity is:

$$v_t = \frac{2mg}{C_D A\rho} \tag{3}$$

We can rearrange this equation to get:

$$C_D = \frac{2mg}{v_t A\rho} \tag{4}$$

Deriving Eq. 1 gives you:

$$C_D = \frac{A_{eff}}{A} \tag{5}$$

Where A_{eff} is the effective cross sectional area and A is the area.

2 Method Experimental Set-up:

For this experiment we used a piece of paper rolled into the shape of a ball, making sure that the inside the ball was hollow. As well as a meter stick to find the initial drop height and a stop watch to measure the time taken for the hollow paper ball to reach the ground.

Before beginning the initial experiment, some data was taken about the type of paper used and the dimensions of the paper ball. In this case the dimensions of the paper used were A4 210 mm x 297 mm and a grammage of 80 g/m^2. We also used a kitchen scales to measure the weight of 9 pages stacked together and took the average weight as the weight of each page, however this method proved to be less precise and the data was discarded for the purpose of this experiment.

Once all of the initial information was taken, the bulk of the experiment took place; it was decided that the initial drop height would be 2 meters and go down in intervals of 10 cm or 0.1 meters, until it was unreasonable to expect our reaction time to the paper ball reaching the ground to be consistent. We dropped the ball from each of these heights, making sure to hit start on the timer at the exact same time that the hollow paper ball was dropped and attempted to stop the timer at the exact point the hollow paper ball reached the ground. We did this 10 times for each of the heights and got an average value. In the end it was possible to measure the drop time for 18 different heights, from 2 meters down to 30 cm, as mentioned above we took 10 measurements of drop time for each of these heights for a total of 180 data points. Only the average times were included in the results section, however the entire table of recorded data is included at the bottom of the report.

Once the data was analysed, the drop time was plotted as a function of the drop height, we used this graph to determine the drag coefficient and its uncertainty.

Figure 2.1: A hollow paper ball made from a 80 g/m^2 A4 sheet.

3 Results:

Using Page Specification:

Page Dimensions: A4 210 mm x 297 mm 80 g/m^2
210 mm x 297 mm = 62,370 mm^2
62,370mm^2 = 0.06237 m^2
80g/m^2 = 80g/m^2 x 0.06237m^2 = $\boxed{4.9896\text{ g}}$ or $\boxed{0.0049896\text{ kg}}$

Using Weighing Scale Method:

weighing scales: 50g per 9 pages.
$\boxed{5.56\text{g}}$ per page or $\boxed{0.00556\text{ kg}}$
Room temperature: 20°C
Air Density: 1.204 kg/m^3

Drop height h_0 (m)	Average drop time t_{avg} (s)
2	1.081
1.9	0.932
1.8	0.843
1.7	0.822
1.6	0.795
1.5	0.778
1.4	0.741
1.3	0.729
1.2	0.709
1.1	0.676
1	0.620
0.9	0.588
0.8	0.549
0.7	0.512
0.6	0.485
0.5	0.444
0.4	0.395
0.3	0.358

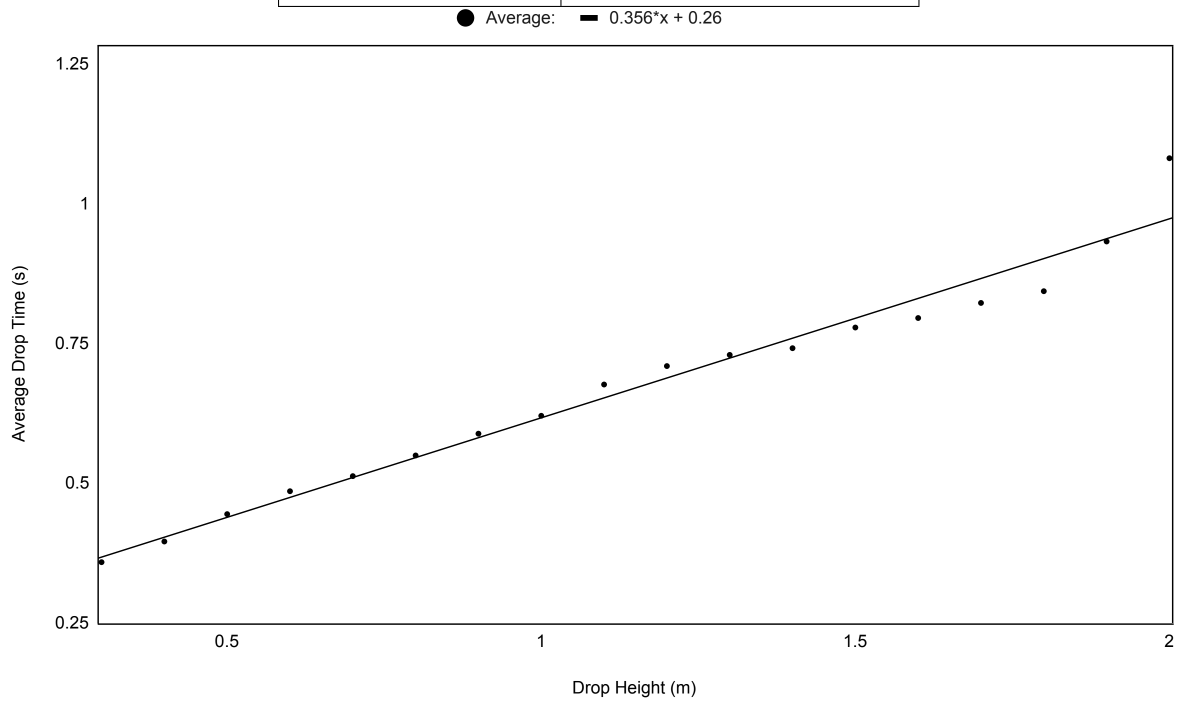

Figure 3.1: Graph of

Using the equation:

$$\frac{y_2 - y_1}{x_2 - x_1} = m \tag{6}$$

We can calculate the slope of the line using two points.
Using Eq. 6:

$$m = \frac{0.62 - 0.512}{1 - 0.7} = 0.36\ s/m\ \pm 0.016 s/m$$

We mentioned **Terminal Velocity** in our introduction, since the slope of our graph is $\frac{Average\ drop\ speed\ (s)}{Drop\ height\ (m)}$ with the units s/m, we can conclude that the velocity of the ball would be the inverse of the slope. We are allowed to make the assumption that the speed is at constant terminal speed over all of the drop distance, so therefore this inverted slope would become our **Terminal Velocity**.

Terminal Velocity:

$$(0.36\ s/m)^{-1} = 2.\dot{7}\ m/s\ \pm 0.11$$

Now we have all of the variables required to work out the drag force using Eq. 4:

$$C_D = \frac{2(0.0049896 kg)(9.81 m/s^2)}{(2.\dot{7}\ m/s)(4\ \pi\ (0.035 m)^2)(1.204 kg/m^3)} = 1.95\ m/s\ \pm 0.04$$

Dimensional Analysis:

$$C_D = \frac{2(\cancel{kg})((\cancel{m/s})(s^{-1}))}{(\cancel{m/s})(m^2)(\cancel{kg}/m^3)} = \frac{s^{-1}}{\frac{m^2}{m^3}} = \frac{s^{-1}}{m^{-1}} = ms^{-1} = \frac{m}{s}$$

Error Analysis: $\frac{\Delta A}{A}$

$\frac{5 \times 10^{-6} kg}{0.0049896 kg} = 0.001$ $\quad \frac{0.11 m/s}{2.\dot{7} m/s} = 0.04$ $\quad \frac{2.5 \times^{-7} m^2}{0.015 m^2} = 1.\dot{6} \times 10^{-5} m^2$ $\quad \frac{0.0005 kg/m^3}{1.204 kg/m^3} = 0.0004$

$$0.001 + 0.04 + 1.\dot{6} \times 10^{-5} + 0.0004 = 0.041416$$

$$\boxed{\text{Terminal Velocity: } 2.\dot{7}\ \text{m/s} \pm 0.11\ \text{m/s}}$$

$$\boxed{\text{Drag Coefficient: } 1.95\ \text{m/s} \pm 0.04\ \text{m/s}}$$

4 Discussion:

While carrying out the experiment a few places where improvements could be made were noted. Similar to the bouncy ball experiment, using human reaction time to determine the time when the hollow paper ball drops and when the hollow paper ball reaches the ground will increase the margin of error of all the results, the same recommendation of using time gates to determine the start and stop times of the hollow paper ball will increase the accuracy of the values greatly.

The purpose of this experiment was to show that sometimes we cannot consider things like drag to be negligible while carrying out an experiment, as we had done previously with the bouncy ball experiment however, in order to determine the drag coefficient of the hollow paper ball we still made the assumption that the speed of the hollow paper ball was at constant terminal speed over the entire drop distance, although the amount of time it would have taken the paper ball to reach this speed might have been negligible, it would still have been better to account for it during the experiment to increase accuracy and to fulfil the overall purpose of the experiment.

5 Conclusion:

The terminal velocity of the hollow paper ball was found to be: $2.\dot{7}$ m/s ± 0.11 m/s
The Drag Coefficient of the hollow paper ball was found to be: 1.95 m/s ± 0.04 m/s
Although the mass and speed of the hollow paper ball was substantially low enough that drag force would affect the system, when comparing this with our bouncy ball experiment involving a much heavier mass falling at a much greater speed we can assume that the drag force was entirely negligible in the overall system.

[Sheet of data]

Drop height	Drop t_1	Drop t_2	Drop t_3	Drop t_4	Drop t_5	Drop t_6	Drop t_7	Drop t_8	Drop t_9	Drop t_{10}
2 m	1.29 s	1.06 s	0.97 s	0.9 s	1.26 s	1.29 s	1.09 s	1 s	1.05 s	0.9 s
1.9 m	1 s	0.8 s	0.95 s	0.95 s	0.87 s	0.96 s	0.98 s	0.9 s	0.9 s	1.01 s
1.8 m	0.75 s	0.8 s	0.86 s	0.86 s	0.8 s	0.87 s	0.9 s	0.87 s	0.82 s	0.9 s
1.7 m	0.79 s	0.81 s	0.79 s	0.88 s	0.8 s	0.84 s	0.87 s	0.78 s	0.82 s	0.84 s
1.6 m	0.83 s	0.76 s	0.81 s	0.8 s	0.8 s	0.79 s	0.77 s	0.79 s	0.8 s	0.8 s
1.5 m	0.8 s	0.8 s	0.79 s	0.79 s	0.85 s	0.7 s	0.74 s	0.74 s	0.78 s	0.79 s
1.4 m	0.6 s	0.72 s	0.73 s	0.79 s	0.8 s	0.8 s	0.74 s	0.7 s	0.73 s	0.8 s
1.3 m	0.74 s	0.75 s	0.76 s	0.7 s	0.81 s	0.68 s	0.69 s	0.65 s	0.71 s	0.8 s
1.2 m	0.7 s	0.67 s	0.61 s	0.74 s	0.75 s	0.74 s	0.68 s	0.69 s	0.8 s	0.71 s
1.1 m	0.65 s	0.67 s	0.66 s	0.75 s	0.63 s	0.74 s	0.74 s	0.65 s	0.62 s	0.65 s
1 m	0.57 s	0.56 s	0.56 s	0.58 s	0.63 s	0.71 s	0.62 s	0.67 s	0.66 s	0.64 s
0.9 m	0.54 s	0.55 s	0.62 s	0.6 s	0.5 s	0.65 s	0.63 s	0.54 s	0.6 s	0.65 s
0.8 m	0.62 s	0.56 s	0.6 s	0.52 s	0.52 s	0.45 s	0.6 s	0.62 s	0.51 s	0.49 s
0.7 m	0.48 s	0.5 s	0.57 s	0.57 s	0.46 s	0.4 s	0.51 s	0.56 s	0.47 s	0.6 s
0.6 m	0.54 s	0.51 s	0.52 s	0.46 s	0.5 s	0.41 s	0.5 s	0.5 s	0.47 s	0.44 s
0.5 m	0.42 s	0.4 s	0.52 s	0.51 s	0.51 s	0.47 s	0.4 s	0.37 s	0.4 s	0.44 s
0.4 m	0.39 s	0.43 s	0.42 s	0.42 s	0.36 s	0.44 s	0.41 s	0.4 s	0.37 s	0.31 s
0.3 m	0.37 s	0.37 s	0.32 s	0.3 s	0.36 s	0.38 s	0.37 s	0.37 s	0.37 s	0.37 s