Density Determination And Error Analysis

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1 Introduction:

Using the equation

$$(P_G(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{(x-\mu)}{\sigma}\right)^2\right] \quad (1)$$

we can work out values for The Gaussian Distribution Function, once we know our values for μ: $\mu = 1.9975$ and σ: $\sigma = 0.04169$.

x	$P_G(x)$	$\Delta P_G(x) = P_G(x)\Delta x$
1.885	0.246	0.00246
1.895	0.458	0.00458
1.905	0.804	0.00804
1.915	1.332	0.01332
1.925	2.083	0.02083
1.935	3.077	0.03077
1.945	4.291	0.04291
1.955	5.650	0.0565
1.965	7.022	0.07022
1.975	8.240	0.0824
1.985	9.128	0.09128
1.995	9.547	0.09547
2.005	9.427	0.09427
2.015	8.788	0.08788
2.025	7.735	0.07735
2.035	6.427	0.06427
2.045	5.041	0.05041
2.055	3.734	0.03734
2.065	2.610	0.0261
2.075	1.723	0.01723
2.085	1.074	0.01074
2.095	0.632	0.00632
2.105	0.351	0.00351
2.115	0.184	0.00184

Table 1: Data for the generation of the Normal Error Function

we can plot our values for $P_G(x)\Delta x$ vs our values for x along a graph

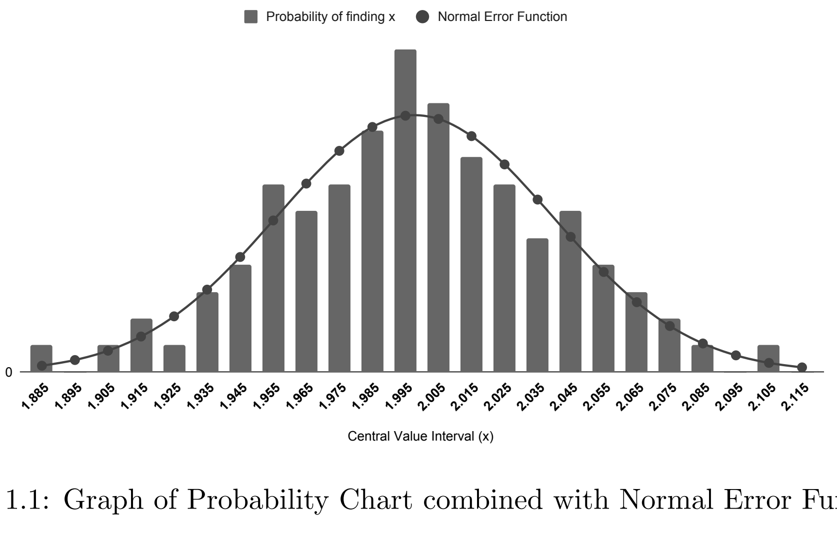

Figure 1.1: Graph of Probability Chart combined with Normal Error Function

1.1 Standard Deviations (σ)

Next we wanted to be able to calculate the number of values that fall between one and two standard deviations of our mean value and represent them on our graph.

In order to calculate the range of values that fall between one standard deviation of our mean we did the simple calculation of adding and subtracting our standard deviation from our mean.

$$\mu \pm \sigma$$
$$1.9975 \pm 0.0417$$
$$Range = 1.9956 - 2.0392$$

We then sum up the frequency of all the values that fall within this range and divide by the total frequency to get our fraction:

$$\frac{7+6+7+9+12+10+8+7+5}{100} = 0.71$$
$$0.71x100 = 71\%$$

We then repeat the steps for 2 standard deviations:

$$\mu \pm 2\sigma$$
$$1.9975 \pm (2)0.0417$$
$$Range = 1.9141 - 2.0809$$

We then sum up the frequency of all the values that fall within this range and divide by the total frequency to get our fraction:

$$\frac{2+1+3+4+7+6+7+9+12+10+8+7+5+6+4+3+2+1}{100} = 0.97$$
$$0.97x100 = 97\%$$

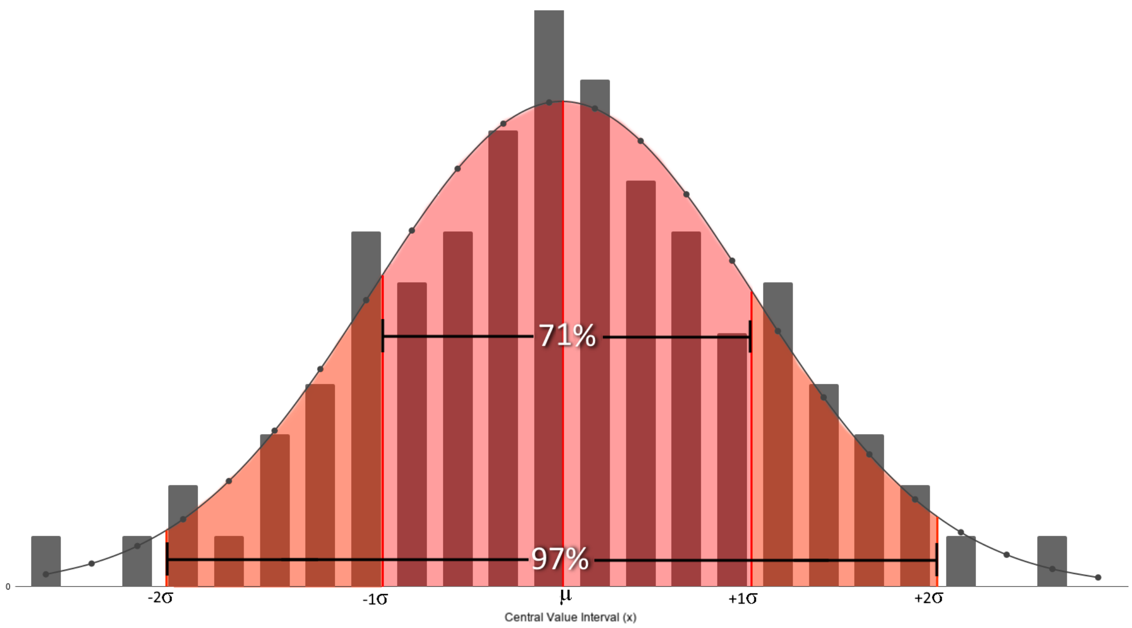

Figure 1.2: Graph of Standard Deviation percentages

2 Results:

Workpiece 1 (copper)			
Dimensions	Vernier Caliper	Micrometer gauge	Electronic-Balance
Length (m)	$(46.00 \pm 0.05)x10^{-3}$	N/A	
Width (m)	$(20.90 \pm 0.05)x10^{-3}$	$(20.439 \pm 0.005)x10^{-3}$	
Height (m)	$(6.10 \pm 0.05)x10^{-3}$	$(5.475 \pm 0.005)x10^{-3}$	
Mass			(0.04859 ± 0.0005) kg

Table 2: Measured dimensions and weight of Workpiece 1

2.1 Volume

As our workpiece 1 was an undetermined metal in the shape of a rectangle we were able to calculate the volume using the formula

$$(Length)x(Width)x(Height) \quad (2)$$

which are represented by (L),(W) and (H) respectively in Fig 2.1. The Calculation was as follows:

$$(46.00x10^{-3})(20.90x10^{-3})(6.10x10^{-3}) = 5.86x10^{-6}m^3 \pm 0.07x10^{-6}m^3$$

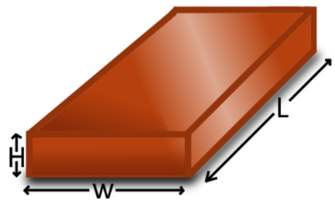

Figure 2.1: A visual representation of Workpiece

Fractional error:

$$\frac{(\Delta L)}{(L)} + \frac{(\Delta W)}{(W)}) + \frac{(\Delta H)}{(H)} = \frac{(\Delta V)}{(V)} \quad (3)$$

$$(\frac{(0.05x10^{-3})}{(46.00x10^{-3})} + \frac{(0.05x10^{-3})}{(20.90x10^{-3})} + \frac{(0.05x10^{-3})}{(6.10x10^{-3})}) = 0.012$$

Percentage error:

$$\frac{(\Delta V)}{(V)}x100 = Percentage\ error \quad (4)$$

$$0.012\,x\,100 = 1.2\%$$

Absolute error:

$$(\Delta V) = \frac{(\Delta V)}{(V)}(V) \quad (5)$$

$$(\Delta V) = 0.012(5.86x10^{-6})$$
$$\Delta V = 0.07x10^{-6}$$

Although we were unable to get a measurement for the Length of the workpiece using a micrometer, we can still calculate a somewhat more accurate volume by substituting in the length measured with a Vernier Calliper making sure to use the Vernier Callipers inaccuracy when calculating our uncertainty

Using Eq. (2):

$$(46.00x10^{-3})(20.439x10^{-3})(5.475x10^{-3}) = 5.15x10^{-6}m^3 \pm 0.01x10^{-6}m^3$$

Using Eq. (3):

$$(\frac{(0.05)}{(46.00)} + \frac{(0.005)}{(20.439)} + \frac{(0.005)}{(5.475)}) = 0.002$$

Using Eq. (4):

$$0.002\,x\,100 = 0.2\%$$

Using Eq. (5):

$$0.002(5.15x10^{-6}) = 0.010x10^{-6}$$

This gave us a slightly more accurate answer with a smaller value of uncertainty, however this could still be improved upon by not crossing instrumentations or recording your measurements using an even more accurate instrument than a micrometer. Because we had to use the vernier calliper uncertainty for one of our calculations, we run into an issue with significant figures, for this reason these measurements will not be used in our density calculations.

2.2 Volumetric Density (ρ)

Since we now have a value for the volume of the workpiece and we also took a measurement of the mass of the workpiece during the experiment, we have all the variables needed in order to be able to calculate the Volumetric Density (denoted by the greek symbol "rho" ρ) Using :

$$\frac{Mass\ of\ Workpiece}{Volume\ of\ Workpiece} \quad (6)$$

Vernier Calliper Measurements Calculation:

$$\frac{0.04859}{5.86x10^{-6}} = 8285.39\,kg\,m^{-3} \pm 182.27\,kg\,m^{-3}$$

Fractional error for volumetric density:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V} \quad (7)$$

$$\frac{0.0005}{0.04859} + 0.012 = 0.022$$

Percentage Error:

$$\frac{\Delta\rho}{\rho}x100 = 2.2\%$$

Absolute Error:

$$0.022\,x\,8285.39 = 182.27$$

It is extremely likely that the unknown metal is Beryllium copper, which has a density of $8100 - 8250\,kg\,m^{-3}$ [1] which falls perfectly within our margins.

Workpiece 2			
Dimensions	Vernier Caliper	Micrometer gauge	Electronic-Balance
Length (m)	$(25.25 \pm 0.05)x10^{-3}$	$(25.015 \pm 0.005)x10^{-3}$	
Diameter(m)	$(25.40 \pm 0.05)x10^{-3}$	$(24.965 \pm 0.005)x10^{-3}$	
Mass			(0.09804 ± 0.0005) kg

Table 3: Measured dimensions and weight of Workpiece 2

2.3 Volume

As our workpiece 2 is an undetermined metal in the shape of a cylinder we are able to calculate the volume using the formula:

$$V = \pi \frac{d^2}{4} h \quad (8)$$

We can get our radius (r) by halving the diameter and our perpendicular height we can assume is equal to the length of the cylinder.

$$\pi((\frac{(25.40x10^{-3})^2)}{4})(25.25x10^{-3}) = 12.79x10^{-6}m^{-3} \pm 0.08x10^{-6}m^{-3}$$

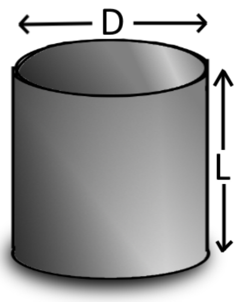

Figure 2.2: A visual representation of Workpiece

Fractional error:

$$2\frac{(\Delta d)}{(d)} + \frac{(\Delta h)}{(h)} = \frac{(\Delta V)}{(V)} \quad (9)$$

$$2\frac{(0.05x10^{-3})}{(25.40x10^{-3})} + \frac{(0.05x10^{-3})}{(25.25x10^{-3})} = 0.006$$

Percentage error:

$$\frac{(\Delta V)}{(V)} x100 = Percentage\ error \quad (4)$$

$$0.006\, x\, 100 = 0.6\%$$

Absolute error:

$$(\Delta V) = \frac{(\Delta V)}{(V)}(V) \quad (5)$$

$$(\Delta V) = 0.006(12.79x10^{-6})$$
$$\Delta V = 0.08x10^{-6}$$

We can repeat these calculations using our measurements from the micrometer to get a more precise answer.

Using Eq. (8):

$$\pi((\frac{(24.965x10^{-3})^2)}{4})(25.015x10^{-3}) = 12.245x10^{-6}m^{-3} \pm 0.007x10^{-6}m^{-3}$$

Using Eq. (9):

$$2\frac{(0.005x10^{-3})}{(24.965x10^{-3})} + \frac{(0.005x10^{-3})}{(25.015x10^{-3})} = 0.0006$$

Using Eq. (4):

$$0.0006x100 = 0.06\%$$

Using Eq. (5):

$$0.0006(12.24x10^{-6}) = 0.007x10^{-6}$$

2.4 Volumetric Density (ρ)

We can reuse the equation we used in workpiece 1 to calculate the density

$$\frac{Mass\,of\,Workpiece}{Volume\,of\,Workpiece} \quad (6)$$

Vernier Calliper Measurements Calculation:

$$\frac{0.09804}{12.79x10^{-6}} = 7665.36\,kg\,m^{-3} \pm 84.32\,kg\,m^{-3}$$

Fractional error for volumetric density:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V} \quad (7)$$

$$\frac{0.0005}{0.09804} + 0.006 = 0.011$$

Percentage Error:

$$\frac{\Delta\rho}{\rho}x100 = 1.1\%$$

Absolute Error:

$$0.011\,x\,7665.36 = 84.32$$

It is likely that the cylinder was made out of either Wrought Iron, which is 7700 $kg\,m^{-3}$ [2] or Stainless steel, which ranges from $7480 - 8000\,kg\,m^{-3}$ [3]

Workpiece 3			
Dimensions	Vernier Caliper	Micrometer gauge	Electronic-Balance
$Length_A$ (m)	$(6.94 \pm 0.05)x10^{-3}$	N/A	
$Length_B$ (m)	$(8.18 \pm 0.05)x10^{-3}$	N/A	
$Length_C$ (m)	$(10.07 \pm 0.05)x10^{-3}$	$(10.160 \pm 0.005)x10^{-3}$	
$Length_D$ (m)	$(13.23 \pm 0.05)x10^{-3}$	$(13.980 \pm 0.005)x10^{-3}$	
$Diameter_A$(m)	$(9.11 \pm 0.05)x10^{-3}$	$(9.641 \pm 0.005)x10^{-3}$	
$Diameter_B$(m)	$(16.32 \pm 0.05)x10^{-3}$	$(16.450 \pm 0.005)x10^{-3}$	
$Diameter_C$(m)	$(21.29 \pm 0.05)x10^{-3}$	$(21.651 \pm 0.005)x10^{-3}$	
$Diameter_D$(m)	$(25.29 \pm 0.05)x10^{-3}$	$(25.410 \pm 0.005)x10^{-3}$	
Mass			(0.09614 ± 0.0005) kg

Table 4: Measured dimensions and weight of Workpiece 3

2.5 Volume

Due to the complexity of this workpiece multiple calculations are required in order to give an accurate representation of the volume. As each section (A,B,C and D) has a different length and diameter, we must calculate their volumes independently. There are also some irregularities which will affect the accuracy of our volume which will be discussed later on in the report.

Using Eq. 8 we can calculate the volume for all four individual sections.

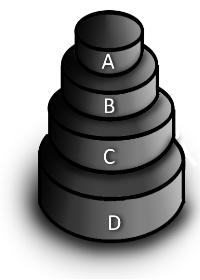

Figure 2.3: A visual representation of Workpiece

Section A:

$$\pi((\frac{(9.11x10^{-3})^2)}{4})(6.94x10^{-3}) = 0.45x10^{-6}m^{-3} \pm 0.008x10^{-6}m^{-3}$$

Section B:

$$\pi((\frac{(16.32x10^{-3})^2)}{4})(8.18x10^{-3}) = 1.71x10^{-6}m^{-3} \pm 0.02x10^{-6}m^{-3}$$

Section C:

$$\pi((\frac{(21.29x10^{-3})^2)}{4})(10.07x10^{-3}) = 3.58x10^{-6}m^{-3} \pm 0.03x10^{-6}m^{-3}$$

Section D:

$$\pi((\frac{(25.29x10^{-3})^2)}{4})(13.23x10^{-3}) = 6.65x10^{-6}m^{-3} \pm 0.05x10^{-6}m^{-3}$$

We can use Eq. (9) to calculate the fractional error of each section

Section A:

$$2\frac{(0.05x10^{-3})}{(9.11x10^{-3})} + \frac{(0.05x10^{-3})}{(6.94x10^{-3})} = 0.01818$$

Section B:

$$2\frac{(0.05x10^{-3})}{(16.32x10^{-3})} + \frac{(0.05x10^{-3})}{(8.18x10^{-3})} = 0.01224$$

Section C:

$$2\frac{(0.05x10^{-3})}{(21.29x10^{-3})} + \frac{(0.05x10^{-3})}{(10.07x10^{-3})} = 0.00966$$

Section D:

$$2\frac{(0.05x10^{-3})}{(25.29x10^{-3})} + \frac{(0.05x10^{-3})}{(13.23x10^{-3})} = 0.00773$$

Percentage Error:

$$0.01818x100 = 1.818\%$$

$$0.01224x100 = 1.224\%$$

$$0.00966x100 = 0.966\%$$

$$0.00773x100 = 0.733\%$$

Absolute Error:

$$(0.01818)(0.45x10^{-6}) = 0.008181x10^{-6}$$

$$(0.01224)(1.71x10^{-6}) = 0.0209304x10^{-6}$$

$$(0.00966)(3.58x10^{-6}) = 0.0345828x10^{-6}$$

$$(0.00773)(6.65x10^{-6}) = 0.0514045x10^{-6}$$

Now that we have worked out our individual values we can add them together to get a representation of the total volume

$$0.45x10^{-6} + 1.71x10^{-6} + 3.58x10^{-6} + 6.65x10^{-6} = 12.39x10^{-6}m^{-3} \pm 0.11x10^{-6}m^{-3}$$

$$0.008x10^{-6} + 0.02x10^{-6} + 0.03x10^{-6} + 0.05x10^{-6} = 0.11$$

2.6 Volumetric Density (ρ)

Now that we have a general representation of the total volume as well as the mass of the object we can calculate the volumetric density using Eq. 6.

$$\frac{0.09614}{12.39x10^{-6}} = 7759.48\, kg\, m^{-3} \pm 892.34\, kg\, m^{-3}$$

Fractional error for volumetric density Eq. 7:

$$\frac{0.0005}{0.09614} + 0.11 = 0.115$$

Percentage Error:

$$\frac{\Delta\rho}{\rho} x100 = 11.5\%$$

Absolute Error:

$$0.115\, x\, 7759.48 = 892.34$$

Metals which fall within this density range include Cast iron which is $6850 - 7750\, kg\, m^{-3}$ [4] or Stainless steel which is $7480 - 7950\, kg\, m^{-3}$ [4].

Although this error is by far the largest one we have, there is actually even more of an inaccuracy within our calculations. The workpiece we used to carry out these measurements had a cylindrical hole located at the base of the piece which would mean that the workpiece is not a completely solid object and thus our volume is off for assuming that it is. The reason the cylindrical hole wasn't taken into consideration during the course of the measurements was it was found to be too difficult to measure using any of the devices available and so it was decided to ignore it during the calculations and note the huge inaccuracy instead. If this measurement was to be repeated to increase accuracy it would be beneficial to have a device that could measure the small cylindrical hole and subtract that from the volume of the base of the workpiece (Section D) represented in Fig. 2.3.

Part 2 Problem 1				
A	b	c	d	f
976.46	5	9	6	$9.76x10^2$
84,200.	5	8	0	$8.42x10^4$
84,200	3	8	2	$8.42x10^4$
0.0094	2	9	4	$9.4x10^{-3}$
301.07	5	3	7	$3.01x10^2$
4.000	1	4	4	4
10.9	3	1	9	$1.09x10$
5280	3	5	8	$5.28x10^3$
414	3	4	4	$4.14x10^2$
0.0308	3	3	8	$3.08x10^{-2}$
30,010	4	3	1	$3x10^4$

Table 5: Part 2 Problem 1 (Table 4)

3 Conclusion:

We were able to conclude the density of workpiece one was: $8285.39\,kg\,m^{-3} \pm 182.27\,kg\,m^{-3}$ and is most likely Beryllium copper, the density of workpiece two was $7665.36\,kg\,m^{-3} \pm 84.32\,kg\,m^{-3}$ and is most likely either Wrought Iron or Stainless steel and workpiece three was $7759.48\,kg\,m^{-3} \pm 892.34\,kg\,m^{-3}$ and is most likely either Cast Iron or Stainless steel.

We also determined that the complexity of the object being measured affects the uncertainty in calculations, so it is best to keep your measurements as small and simple as possible in order to achieve the most accurate result.

4 References:

[1] "Metals and Alloys - Densities - The Engineering Toolbox.", https://bit.ly/2yQ4Vf0 Accessed 4 Oct. 2019.

[2] "Wrought Iron - Properties, Applications", https://www.azom.com/article.aspx?ArticleID=9555 Accessed 6 Oct. 2019

[3] "Metals and Alloys - Densities - The Engineering Toolbox.", https://bit.ly/2yQ4Vf0 Accessed 6 Oct. 2019.

[4]"Density of metals and elements table", https://bit.ly/2Os2CHR Accessed 7 Oct. 2019